Δ-OPEN SETS AND δ-SEMI CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

P. Piriyalucksan and M. Arunmaran*

Department of Mathematics and Statistics, University of Jaffna, Sri Lanka

The study of fuzzy topological spaces was introduced by Chang in 1968, following the discovery of fuzzy sets by Zadeh. So far, several types of open sets and closed sets in fuzzy topological spaces have been documented in the literature. Some of them are fuzzy pre-open sets, fuzzy α-open sets, fuzzy β-open sets, fuzzy regular open sets, and fuzzy semi-open sets. In this abstract, we describe two new types of sets, namely fuzzy Δ-open sets and fuzzy δ-semi-closed sets, and discuss the properties of these sets. For a non-empty set X, a fuzzy topology is a family τ of fuzzy subsets in X satisfying the following conditions: $0_X , 1_X \in \tau$; the finite intersection of members of τ is a member of τ; and the arbitrary union of members of τ is a member of τ. We call the pair (X, τ) a fuzzy topological space. Also, the elements of τ are called fuzzy open sets. First, we define fuzzy Δ-open set in fuzzy topological spaces. A subset D of a fuzzy topological space X is called fuzzy Δ-open if $D = (A \wedge B^C) \vee (B \wedge A^C)$, where A and B are fuzzy open sets. When the set D is called fuzzy semi Δ-open, the above equation holds with the fuzzy semi-open sets A and B. Similarly, we can define the following sets: fuzzy pre Δ-open / fuzzy α-Δ-open / fuzzy β-Δ-open. Next, we show that every fuzzy Δ-open set is fuzzy semi Δ-open. However, the converse need not be true in general. Next, we define fuzzy δ-open sets in fuzzy topological spaces. A subset A is called fuzzy δ-open if for every $x \in A$, there exists a regular open set G such that $x \in G \leq A$. A subset A is called fuzzy δ-semi closed if there exists a fuzzy δ-closed set F such that $int(F) \leq A \leq F$. Next, we show the following: A subset A is called fuzzy δ-semi open if and only if $X \backslash A$ is fuzzy δ-semi closed; every fuzzy δ-closed set is fuzzy δ-semi closed; every fuzzy δ-semi closed set is fuzzy semi-closed; a fuzzy set A is fuzzy δ-semi closed if and only if $int(\delta cl(A)) \leq A$. Finally, we show that the intersection or union of two fuzzy δ-semi closed sets is also fuzzy δ-semi closed.

Keywords: fuzzy topology, fuzzy Δ-open set, fuzzy δ-semi closed

Corresponding Author: marunmaran03@gmail.com

Δ-OPEN SETS AND δ-SEMI CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

P. Piriyalucksan and M. Arunmaran*
Department of Mathematics and Statistics, University of Jaffna, Sri Lanka

INTRODUCTION

The concept of fuzzy topological spaces was first introduced by Chang [1] in 1968, after he was motivated by the discovery of fuzzy sets that were introduced by Zadeh [2]. Chang considered the essential notions of classical topology to fuzzy sets in the development of a new theory of fuzzy topology. Following that, several authors have expanded the study on fuzzy topology to new types of open sets, closed sets. Various types of open and closed sets in fuzzy topological spaces were already documented in the literature. Some of them are fuzzy pre-open sets, fuzzy α-open sets, fuzzy β-open sets, fuzzy regular open sets, and fuzzy semi-open sets. In this paper, we study two new kinds of sets, namely Δ-open sets and δ-semi closed sets, and obtain the properties of these sets.

Now, we explain the concept of fuzzy sets. A fuzzy subset of a set X is expressed as $\{(x, \mu_A(x)) : x \in X\}$, where $\mu_A: X \longrightarrow [0,1]$ is the membership function. Also, a fuzzy subset A in X is defined as a function from A into the unit interval $[0,1]$. Moreover, two fuzzy subsets A and B in X are compared as $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$. In addition, the membership function of the intersection of two fuzzy sets A and B is defined by $A \wedge B = min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$. Similarly, the membership function of the union of two fuzzy sets A and B is defined by $A \vee B = max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.

A fuzzy subset A in the set X is empty if and only if its membership function is zero on X, and is denoted by 0_X . Similarly, the set X is a fuzzy subset of X whose membership function is 1 on X, and is denoted by 1_X. Also, the complement of a fuzzy set A in X is denoted by $1 - A$.

For a non-empty set X, a fuzzy topology is a family τ of fuzzy subsets in X satisfying the following conditions: $0_X , 1_X \in \tau$; the finite intersection of members of τ is a member of τ; and the arbitrary union of members of τ is a member of τ. We call the pair (X, τ) a fuzzy topological space. Also, the elements of τ are called fuzzy open sets. Moreover, the complement of fuzzy open sets is fuzzy closed sets.

METHODOLOGY

In this section, we provide some useful definitions that are needed for this paper.

Definition: Let (X, τ) be a fuzzy topological space, and A be a fuzzy set. Then the set A is called fuzzy α-open if $A \leq int\left(cl\left(int(A)\right)\right)$; fuzzy semi-open if $A \leq cl\left(int(A)\right)$; fuzzy pre-open if $A \leq int\left(cl(A)\right)$; fuzzy β-open if $A \leq cl(int(cl(A)))$; fuzzy regular-open if $A = int\left(cl(A)\right)$.

RESULTS AND DISCUSSION

In this section, we provide our results related to Δ-open sets and δ-semi closed sets in fuzzy topological spaces. A subset D of a fuzzy topological space X is called fuzzy Δ-open if $D = (A \wedge B^C) \vee (B \wedge A^C)$, where A and B are fuzzy open sets. When the set D is called fuzzy semi Δ-open, the above equation holds with sets A and B that are fuzzy semi-open. Similarly, we can define the following sets: fuzzy pre Δ-open/ fuzzy $\alpha - \Delta$-open / fuzzy $\beta - \Delta$-open.

For example, for the set $X = \{A, B, C\}$, we define two fuzzy sets $A = \{< A, 0.1 >, < B, 0.5 >, < C, 0.3 >\}$, $B = \{< A, 0.3 >, < B, 0.4 >, < C, 0.6 >\}$. Then, we can find two fuzzy semi open sets $F = \{< A, 0.6 >, < B, 0.5 >, < C, 0.3 >\}$ and $E = \{< A, 0.8 >, < B, 0.4 >, < C, 0.6 >\}$. Then, $G = \{< A, 0.4 >, < B, 0.5 >, < C, 0.6 >\}$ is fuzzy semi Δ-open set.

Next, we show that every fuzzy Δ-open set is fuzzy semi Δ-open. Let G be a fuzzy Δ-open set. Then there exist two fuzzy open sets A, B such that $G = (A \wedge B^c) \vee (B \wedge A^c)$. But every fuzzy open set is fuzzy semi-open. Then A and B are fuzzy semi-open sets. Then G is fuzzy semi Δ-open sets. However, every fuzzy semi-Δ-open set need not be fuzzy Δ-open. Because a fuzzy semi-open set is not a fuzzy open set. In this way, we obtain similar results. That is, every fuzzy Δ-open set is fuzzy $\beta - \Delta$-open, every fuzzy $\alpha - \Delta$-open set is fuzzy $\beta - \Delta$-open, every fuzzy pre $-\Delta$-open set is fuzzy $\beta - \Delta$-open, every fuzzy semi-Δ-open set is fuzzy $\beta - \Delta$-open. However, the converse of these results need not be true.

Next, we discuss the fuzzy δ-open sets in fuzzy topological spaces. A subset A is called fuzzy δ-open if for every $x \varepsilon A$, there exists a regular open set G such that $x \varepsilon G \leq A$. A subset A is called fuzzy δ-semi-open if there exists a fuzzy δ-open set U such that $U \leq A \leq cl(U)$. A subset A is called fuzzy δ-semi-closed if there exists a fuzzy δ-closed set F such that $\text{int}(F) \leq A \leq F$.

Now, we show that a subset A is called fuzzy δ-semi-open if and only if $X \backslash A$ is fuzzy δ-semi-closed. To prove this result, we pick a fuzzy δ-semi-open set A. Then there exists a fuzzy δ-open set U such that $U \leq A \leq cl(U)$. This implies that $int(U^c) \leq A \leq U^c$. Put $U^c = V$. Then V is a fuzzy δ-closed set and $int(V) \leq X \backslash A \leq V$. Thus, $X \backslash A$ is fuzzy δ-semi-closed. Conversely, let $X \backslash A$ is fuzzy δ-semi-closed. Then $int(F) \leq X \backslash A \leq F$, for some fuzzy δ-closed set F. This implies that $F^c \leq A \leq cl(F^c)$ and F^c is fuzzy δ-open set. Thus, A is fuzzy δ-semi-open.

Next, we show that every fuzzy δ-closed set is fuzzy δ-semi closed. Pick a fuzzy δ-closed set A. Then $A = \delta cl(A)$. But $int(A) = int(\delta cl(A)) \leq A$. Hence, A is δ-semi-closed. Similarly, every fuzzy δ-semi-closed set is fuzzy semi-closed. Next, we show that a fuzzy set A is fuzzy δ-semi-closed if and only if $int(\delta cl(A)) \leq A$. To prove this, assume that $int(\delta cl(A)) \leq A$. Then there exists a fuzzy δ-closed set $F = \delta cl(A)$ such that $int(F) \leq A \leq F$. Thus, A is fuzzy δ-semi-closed. Conversely, let A be fuzzy δ-semi-closed. Let G be fuzzy δ-closed set. Then $int(G) \leq A \leq G$. Thus, $int(\delta cl(A)) \leq A$. Finally, we show that the intersection of two fuzzy δ-semi-closed sets is also fuzzy δ-semi-closed. To show this, we pick two fuzzy δ-semi-closed sets A and B. Then there exist fuzzy δ-closed sets U and V such that $int(U) \leq A \leq U$ and $int(V) \leq B \leq V$. Then $int(U) \wedge int(V) = int(U \wedge V) \leq A \wedge B \leq U \wedge V$. Thus, $A \wedge B$ is fuzzy δ-semi-closed set. Similarly, we can show that the union of two fuzzy δ-semi-closed sets is also fuzzy δ-semi-closed.

CONCLUSIONS/RECOMMENDATIONS

In this paper, we have described new types of fuzzy open and closed sets in fuzzy topological spaces. We have explained different kinds of fuzzy Δ-open sets and some of their properties. Also, we have discussed fuzzy δ-open sets and fuzzy δ-semi-open/closed sets and addressed some of their properties.

REFERENCES

[1] Zadeh, L. A., Fuzzy sets. (1965). *Information and Control*, 8, 338-353.

[2] Chang, C. L., Fuzzy topological spaces (1968), *J. Math. Anal. Appl.* 24, 182-190.