MATHEMATICAL MODELLING FOR FINGERO-IMBIBITION PHENOMENON INFLUENCE OF THE MAGNETIC FIELD DURING DIFFERENT NANO FLOODINGS

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Thermal, chemical, and flooding methods of enhanced oil recovery have been used to increase the production of residual oil in reservoirs. In recent years, some studies have investigated new strategies such as injecting nano-fluid, boosting performance using new technologies, and incorporating magnetic fields, to improve oil recovery. During the Enhanced Oil Recovery (EOR) process, the fingero-imbibition phenomenon occurs when oil filled porous medium encounters another phase that preferentially wets the medium and the wetting phase flows into the medium while the native phase flows out. The event caused by the varying wetting capacities of the phases is referred to as the imbibition phenomenon. Furthermore, if a porous medium filled with one phase is displaced by another phase of lower viscosity rather than its regular displacement of the entire front, protuberances may form as a result of the fluid injection through the porous medium at a relatively high speed, causing the fingering phenomenon. In this study, we developed a new mathematical model to determine the injective fluid saturation of the fingero-imbibition phenomenon while accounting for the influence of the magnetic field and utilizing several nano-powders (aluminum oxide, magnesium oxide, and silicon dioxide) as injective fluid for an inclined oil layer through a homogeneous porous medium. To solve the model, which is a nonlinear partial differential equation, the Method of Directly Diffing the Inverse Mapping was used. Using the obtained results, we can see that the saturation of nano-water of fingero-imbibition phenomenon increases along with the distance as well as the inclination angle for a fixed time. The saturation of injected fluid increases due to the magnetic field effect, which is greater than it is without the magnetic field effect. The results show that the brine mixed with aluminum oxide has the maximum saturation compared to the others, and the mixture with magnesium oxide has the lowest saturation. We can conclude that the brine with aluminum oxide benefits EOR since the oil recovery factor is directly proportional to the saturation of the injective fluid.

Keywords: Enhanced oil recovery, Gravitational effect, Magnetic field, Method of Directly Diffing the inverse Mapping, Nano powders

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INTRODUCTION

Injection of nanofluids into the oil reservoirs is a recent approach to chemical flooding in the tertiary process (Khademolhosseinia, 2015) in Enhanced Oil Recovery (EOR). Unlike conventional EOR techniques used in the primary and secondary stages, this approach has the potential to produce an additional amount of oil. Aluminum oxide, magnesium oxide, silicon dioxide, carbon nanotubes, bacterial cellulose nanocrystals, graphene oxide, and clay materials are some examples of materials that can be potentially used in nanoflooding. In addition, thermal flooding and miscible flooding can also use in the tertiary stage. This study was carried out to find a better one-dimensional nanoparticle for EOR out of aluminum oxide, magnesium oxide, and silicon dioxide, by dissolving 0.4% of each nanopowder into the brine (1000g of water + 30g of salt) separately when a magnetic field appears. The main objective of this study is to present a new mathematical model to determine the saturation of nanofluids in the finger-imbibition phenomenon for the inclined oil layer in the presence of a magnetic field and compare it with the experimental results. When a porous medium filled with one phase (oil) comes into contact with another phase (nano brine) that preferentially wets the medium, the wetting phase (nano brine) flows into the medium and the native phase (oil) flows out. Imbibition phenomenon is the name given to this occurrence that results from the different wetting capacities of the phases. This phenomenon together with the fingering phenomenon which occurs due to the viscosity difference of two phases is known as fingero-imbibition phenomenon. Second-order approximate solutions for saturations of nanofluids for inclination angles 0^0 and 10^0 were obtained when the least squared residual error occurs using the Method of Directly Defining the inverse Mapping (MDDiM) which is a novel technique to solve nonlinear differential equations. It was first introduced by Liao in 2016 (Liao, 2016) and it was extended by Dewasurendra et al. to solve a system of coupled nonlinear ordinary differential equations (Dewasurendra, 2018). In 2021, Sahabandu et al. further extended this novel technique to solve nonlinear partial differential equations (Sahabandu, 2021). The resultant graph which was obtained using Maple 16 shows the highest saturation from aluminum oxide fluid compared to others while magnesium oxide gives the lowest saturation when a magnetic field is present. Since the oil recovery factor is directly proportional to the saturation of injective fluid, we can conclude that using aluminum oxide provides a great benefit in EOR.

METHODOLOGY

Seepage velocities of fluid can be represented using Darcy's law as (Scheidegger, 1960), (Bear, 1972);

$$V_i = -\frac{Kk_i}{\mu_i}\left[\frac{\partial P_i}{\partial x} - \rho_i g \ \sin\alpha + \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right] \tag{1}$$

$$V_n = -\frac{Kk_n}{\mu_n}\left[\frac{\partial P_n}{\partial x} - \rho_n g \ \sin\alpha\right] \tag{2}$$

Where V_i, V_n - seepage velocity, k_i, k_n- relative permeability, μ_i, μ_n- constant viscosity, ρ_i, ρ_n-density, P_i, P_n-pressure and S_i, S_n- saturation of water–nano brine mixture and oil–nano brine mixture

respectively, g- gravitational acceleration, K- permeability, μ is permeability of magnetic field H and, x- distance, t-time, α- angle of the incline.
Effect from capillary pressure P_c,

$$P_c = P_n - P_i \tag{3}$$

At the imbibition face, $V_n = -V_i$ (4)

By (4) and (3)

$$-\frac{Kk_n}{\mu_n}\left[\frac{\partial P_n}{\partial x} - \rho_n g \sin\alpha\right] = \frac{Kk_i}{\mu_i}\left[\frac{\partial P_i}{\partial x} - \rho_i g \sin\alpha + \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right]$$

$$\frac{\partial P_i}{\partial x} = \frac{\left(\frac{k_n}{\mu_n}\left(\rho_n g \sin\alpha - \frac{\partial P_c}{\partial x}\right) + \frac{k_i}{\mu_i}\left(\rho_i g \sin\alpha - \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right)\right)}{\left(\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right)} \tag{5}$$

By (5) in (1) we get,

$$V_i = -\frac{Kk_i}{\mu_i}\left[\frac{\left(\frac{k_n}{\mu_n}\left(\rho_n g \sin\alpha - \frac{\partial P_c}{\partial x}\right) + \frac{k_i}{\mu_i}\left(\rho_i g \sin\alpha - \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right)\right)}{\left(\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right)} - \rho_i g \sin\alpha + \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right]$$

$$V_i = -\frac{Kk_i}{\mu_i}\left[\frac{\left(\frac{k_n}{\mu_n}\left(\rho_n g \sin\alpha - \rho_i g \sin\alpha - \frac{\partial P_c}{\partial x} + \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right)\right)}{\left(\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right)}\right] \tag{6}$$

$$\frac{k_i}{\mu_i}\frac{k_n}{\mu_n}\left(\frac{k_i}{\mu_i} + \frac{k_n}{\mu_n}\right)^{-1} \approx \frac{k_n}{\mu_n} \tag{7}$$

By (6) and (7)

$$V_i = \frac{Kk_n}{\mu_n}\left((\rho_i - \rho_n)g \sin\alpha + \frac{\partial P_c}{\partial x} - \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right) \tag{8}$$

By mass balance,

$$P\frac{\partial S_i}{\partial t} + \frac{\partial V_i}{\partial x} = -\frac{U}{\rho_i \omega_i^{NP}}, \tag{9}$$

$$P\frac{\partial S_n}{\partial t} + \frac{\partial V_n}{\partial x} = -\frac{U}{\rho_n \omega_n^{NP}}\ . \tag{10}$$

Here, $\omega_i^{NP}, \omega_n^{NP}$ - mass fraction of the nano powder component in the water–nano brine mixture and oil–nano brine mixture respectively, P-porosity of homogeneous porous medium and U- mass transfer term.

By (8) and (9),

$$P\frac{\partial S_i}{\partial t} + \frac{K}{\mu_n}\frac{\partial}{\partial x}k_n\left((\rho_i - \rho_n)g \sin\alpha + \frac{\partial P_c}{\partial x} - \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right) = -\frac{U}{\rho_i \omega_i^{NP}}.$$

Due to phase saturation we get, $S_i + S_n = 1,\ k_i = S_i,\quad k_n = S_n = 1 - S_i$.
Assuming capillary pressure as a linear function of saturation $P_c = -\beta S_i$; β is the capillary pressure coefficient.

$$P\frac{\partial S_i}{\partial t}+\frac{K}{\mu_n}\frac{\partial}{\partial x}\left((1-S_i)\left((\rho_i-\rho_n)g\ \sin\alpha+\frac{\partial(-\beta S_i)}{\partial x}-\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right)\right)=-\frac{U}{\rho_i\omega_i^{NP}}.$$

Define, $U=K_0\left(C_i^{NP}-k_{eq}C_n^{NP}\right), k_{eq}=\frac{C_n^{NP}}{C_i^{NP}}$; $\quad C_\alpha^{NP}=\omega_\alpha^{NP}\rho_\alpha;\ \ \alpha=i,n$

This implies, $U=\frac{K_0}{C_i^{NP}}\left(\left(C_i^{NP}\right)^2-(C_n^{NP})^2\right)$. where K_0 −mass transfer coefficient, k_{eq}-distribution or partition coefficient of nano powder and C_i^{NP}, C_n^{NP}-nano powder concentration in water and oil phases.

$$P\frac{\partial S_i}{\partial t}+\frac{K}{\mu_n}\frac{\partial}{\partial x}\left((1-S_i)\left((\rho_i-\rho_n)g\ \sin\alpha+\frac{\partial(-\beta S_i)}{\partial x}-\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right)\right)=-\frac{K_0}{(C_i^{NP})^2}((C_i^{NP})^2-(C_n^{NP})^2)$$

Considering the magnetic field in the x- direction only, we write H as $H=\lambda x^n$, where λ is a constant parameter and n is an integer. When $n=1$,

$$P\frac{\partial S_i}{\partial t}+\frac{K}{\mu_n}\frac{\partial}{\partial x}\left((1-S_i)\left((\rho_i-\rho_n)g\ \sin\alpha-\beta\frac{\partial S_i}{\partial x}-\frac{\mu\lambda x}{4\pi}\frac{\partial(\lambda x)}{\partial x}\right)\right)=-K_0\left(1-{k_{eq}}^2\right).$$

$$P\frac{\partial S_i}{\partial t}-\frac{K\beta}{\mu_n}\frac{\partial}{\partial x}\left((1-S_i)\frac{\partial S_i}{\partial x}\right)-\frac{K\mu\lambda^2}{4\pi\mu_n}\frac{\partial}{\partial x}((1-S_i)x)+\frac{K(\rho_i-\rho_n)g\ \sin\alpha}{\mu_n}\frac{\partial}{\partial x}(1-S_i)=K_0\left({k_{eq}}^2-1\right).$$

Let $X=\frac{x}{L}$, $\quad T=\frac{K\beta\, t}{\mu_n L^2 P}$, $\ 1-S_i(x,t)=S_i(X,T)$ where X and T are dimensionless parameters of distance and time respectively.

$$\frac{K\beta P}{\mu_n L^2 P}\frac{\partial S_i}{\partial T}-\frac{K\beta}{\mu_n L}\frac{\partial}{\partial X}\left(\frac{S_i}{L}\frac{\partial S_i}{\partial X}\right)-\frac{K\mu\lambda^2}{4\pi\mu_n L}\frac{\partial}{\partial X}(S_i XL)+\frac{K(\rho_i-\rho_n)g\ \sin\alpha}{\mu_n L}\frac{\partial}{\partial X}S_i=K_0\left({k_{eq}}^2-1\right)$$

$$\frac{\partial S_i}{\partial T}-\frac{\partial}{\partial X}\left(S_i\frac{\partial S_i}{\partial X}\right)-A\sin\alpha\frac{\partial}{\partial X}(S_i X)+B\frac{\partial}{\partial X}S_i-C=0 \qquad (11)$$

Where $A=\frac{L^2\mu\lambda^2}{4\pi\beta}$ and $\quad B=\frac{(\rho_i-\rho_n)gL}{\beta}$, $\ C=\frac{\mu_n L^2 K_0}{K\beta}\left({k_{eq}}^2-1\right)$.

Equation (11) is the governing equation of finger-imibibibtion phenomenon when magnetic field is present for miscible fingers. To solve the above model, we considered the initial condition as, $S_i(X,0)=0.01X^2$. Now, consider the n^{th}-order nonlinear differential equation $N\,[u(x)]=0$, and the equation of MDDiM,

$$u_k=\chi_k\,u_{k-1}+hL^{-1}[\delta_{k-1}]+\sum_{n=1}^{\mu}a_{k,n}\phi_n. \qquad (12)$$

Defined $\chi_k=\begin{cases}0, & k\le 1\\ 1, & k>1\end{cases}$

Here, L^{-1} is the inverse liner mapping, N is the nonlinear part, $a_{k,n}$ is real constant, ϕ_n is base function, and h is the convergence control parameter, which should be determined. By applying MDDiM to equation (11) we obtained equation (13) (Sahabandu, 2021).

$$S_{i_k}=\chi_k\,S_{i_{k-1}}+hL^{-1}[\delta^k]+a_{k,0}\,,\ \text{for}\ k\ \ge 1. \qquad (13)$$

By considering,

$$N\,[s_i(X,T)]=\frac{\partial S_i}{\partial T}-\frac{\partial}{\partial X}\left(S_i\frac{\partial S_i}{\partial X}\right)-A\sin\alpha\frac{\partial}{\partial X}(S_i X)+B\frac{\partial}{\partial X}S_i-C=0, \qquad (14)$$

with initial condition, we obtained an initial guess as $S_{i_0}(X,T)=0.01X^2$. In the frame of MDDiM we have great freedom to choose an inverse linear mapping. For this paper, we chose,

$$L^{-1}[T^k]=\frac{T^{k+1}}{Dk+1}. \qquad (15)$$

Here, D is an arbitrary constant, and the following results are obtained using Maple 16.

The introduction of a magnetic field can introduce additional complexities and uncertainties to the modeling and production of fingering phenomena in the oil industry. When a magnetic field is introduced, several factors come into play that can impact the accuracy and predictability of mathematical models. Complex fluid behavior, magnetohydrodynamics (MHD) effects, nonlinear effects, anisotropic permeability, electromagnetic properties, scale dependence, limited experimental data, numerical challenges, and parameter sensitivity are some uncertainties and challenges associated with modelling fingering phenomena in the presence of a magnetic field. Addressing these uncertainties requires a multidisciplinary approach that combines fluid dynamics, electromagnetics, and porous media physics, advanced numerical simulation techniques, experimental validation, and sensitivity analysis are crucial for improving the accuracy of models and predictions when magnetic fields are involved in fingering phenomena within the oil industry.

RESULTS AND DISCUSSION

By setting $A = 3.979 \times 10^{-9}, B = -0.0015(Al_2O_3), -0.028(SiO_2), -0.057(MgO)$, $C = 1.042(Al_2O_3), 1.038(SiO_2), 1.002(MgO)$ and $D = 1$ we obtained following tables and graphs for second and second- order approximate solutions.

For order 2	$\alpha = 0^0$		$\alpha = 10^0$	
NP	h	E(h)	h	E(h)
Al_2O_3	-0.33683	2.1478×10^{-3}	-0.33984	9.0124×10^{-4}
SiO_2	-0.33683	2.1409×10^{-3}	-0.33984	9.0945×10^{-4}
MgO	-0.33683	2.0118×10^{-3}	-0.33984	8.5462×10^{-3}

Table 1: Corresponding squared residual errors $E(h)$ and values of converge control parameter h.

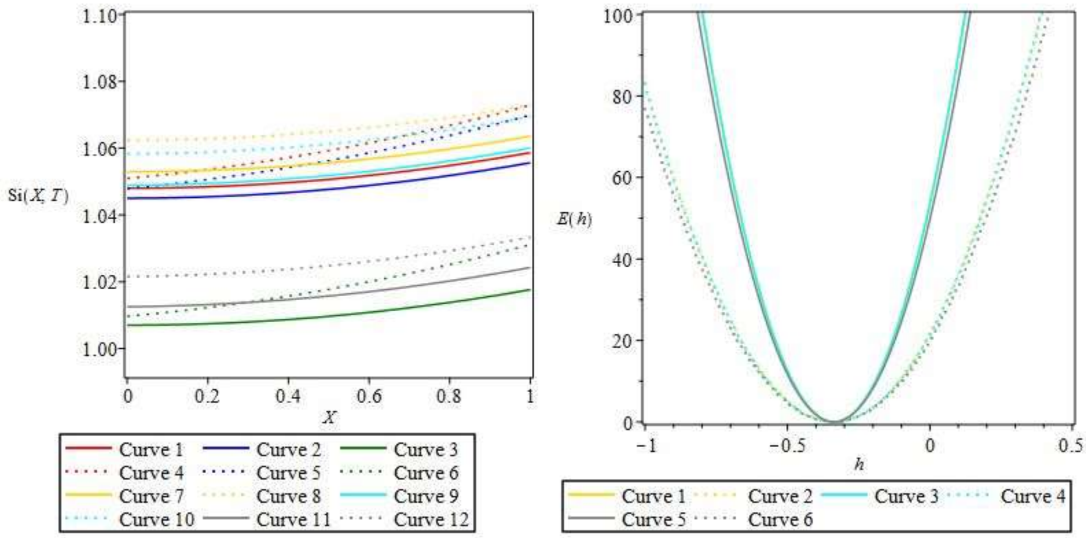

Figure 1: Saturation of nanofluid $S_i(X,T)$ versus distance X for fixed value of time T=1 (left) and corresponding error graphs (right). (Solid and dash lines represent inclination angles $\alpha = 0^0$ and $\alpha = 10^0$ and $Al_2O_3 -$ red and yellow, $SiO_2 -$ blue and cyan, and $MgO -$ green and gray , and curves 1-6 without magnetic field, curves 7-12 presence of magnetic field represent respectively).

The saturation of nanofluids increases with the inclination angle and as well as the distance $\boldsymbol{X}$ for a fixed time $\boldsymbol{T = 1}$, as shown in figure 1. We can observe that the saturation of nanofluids is higher when magnetic field is present than when it is absents (see figure 1 right). For both cases (when magnetic field presents and when it absents) higher saturation is given by $\boldsymbol{Al_2O_3}$ while lower saturation is given by $\boldsymbol{MgO}$. These results agreed with the experimental results of Odo, 2021 (Odo, 2021).

CONCLUSIONS/RECOMMENDATIONS

We obtained three terms approximate solution for the nonlinear PDE using Maple 16 by applying the MDDiM technique. All the solutions are accurate enough, with the squared residual errors as given in table 1. The saturation of nanofluids increases with the inclination angle and the distance X for a fixed time $T = 1$, as shown in figure 1. We can observe that the saturation of nanofluids is higher when a magnetic field is present than when it is absent (see figure 1 right). Although several studies have demonstrated the benefits of a magnetic field in the immiscible fingero-imbibition phenomenon (Patel, 2017), this study concludes for the first time that the magnetic field effect plays an important role in the miscible fingero-imbibition phenomenon. As the oil recovery factor is directly proportional to the saturation of injective fluid, the result in figure 1 shows that Al_2O_3 leads to better outcomes than the other two nanoparticles. The second-best option is the SiO_2, followed by MgO. We compared our mathematical findings with the experimental results (Odo, 2021) and they were found to be consistent.

REFERENCES

Bear, J., (1972), Dynamics of fluids in porous media, *American Elsevier publishing company*, inc., New York.

Dewasurendra M, Baxter M, Vajravelu K. (2018) *A Method of Directly Defining the Inverse Mapping for Fourth Order Non-Linear Systems Arising in Combined Free and Forced Convection in a Second Grade Fluid.* Applied Mathematics and Computation 339. 758-767.

Dewasurendra M, Vajravelu K. (2018) *On the Method of Inverse Mapping for Solutions of Coupled Systems of Nonlinear Differential Equations Arising in Nanofluid Flow, Heat and Mass Transfer.* Applied Mathematics and Nonlinear Science *3. 1-14.*

Khademolhosseinia R, Jafaria A, Shabania MH. (2015) *Micro Scale Investigation of Enhanced Oil Recovery Using Nano/Bio Materials.* Procedia Materials Science 11. *171 – 175.*

Liao S, Zhao Y. (2016) *On the method of directly defining inverse mapping.* Numer Algor.

Odo J. E., Onyejekwe I., Anthony C. et al. (2021) *Effect of the Application of Aluminium oxide on Recovery in Enhanced Oil Recovery*, Engineering and Applied Sciences, 6(2): 41-48.

Sahabandu C.W., Karunarathna D., Sewvandi P., Juman Z.A.M.S., Dewasurendra M., and Vajravelu K. (2021) *A Method of Directly Defining the inverse Mapping for a nonlinear partial differential equation and for systems of nonlinear partial differential equations.* Computational and Applied Mathematics. 40.6, 1-16.

Scheidegger, A. E., (1960) The physics of flow through porous media, revised edition, *University of Toronto press,* Toronto.