AN INTEGRATED MODEL OF THE CAPACITATED VEHICLE ROUTING PROBLEM AND THE VEHICLE SCHEDULING PROBLEM AT THE MULTI-DOOR DEPOT

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The goal of an efficient supply chain (*SC*) is to supply or deliver the shipments to the right place in the right quantity at the right time with a low cost. To be an efficient *SC*, the coordination and integration of the activities in the *SC* are mandatory. Routing vehicles to collect the shipments from the suppliers with minimum travelling cost is an optimization problem in the *SC*. Once the shipments are collected from the suppliers, the routed vehicles must return to the depot which generally has *multi-doors*. When the doors at the depot are limited and busy, the returned vehicles have to wait to unload the accumulated shipments. Therefore, properly coordinating and scheduling these vehicles to those doors at the depot to minimize the waiting time is considered to be an optimization problem in the *SC*. Therefore, in this study, *routing vehicles* to collect the shipments from suppliers and *scheduling vehicles* to doors at the depot, based on *first come first serve* basis, are simultaneously solved. Hence, the objective of this integrated vehicle routing and scheduling problem (*VR&SP*) is to minimize the total cost which contains the following components: *vehicle travelling cost* between suppliers, *loading cost* at the suppliers, *vehicle waiting cost*, *unloading cost* at the depot and *vehicle operations cost*. A mixed integer quadratic programming (*MIQP*) model is developed to solve the integrated *VR&SP*. The *Branch and Bound* algorithm is employed to obtain the exact solution to this *MIQP* using *LINGO* optimization software. Since *LINGO* is not capable of handling large-scale instances, only the small-scale instances are taken into account. The input data are generated randomly and the compatibility of the developed model is verified by numerical illustrations. Therefore, it can be concluded that this model solves the vehicle routing to suppliers and vehicle scheduling to doors simultaneously. Since *VR&SP* is a *NP*-hard problem, heuristics or meta-heuristic methods are proposed to solve the large-scale instances. Furthermore, this *VR&SP* model can be extended to vehicle routing with cross-docking (*CD*) problem. Therefore, it is recommended to attempt an integrated model combining *VR&SP* with *CD* as a future study.

Key words: Multi-door depot, vehicle routing, vehicle scheduling.

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1. INTRODUCTION

The significance of the supply chain (SC) plays an important role in the competitive environment of the global market. Then the goal of the efficient SC is that the products must be supplied and delivered to the right place in the right quantity at the right time with a low cost. Therefore, to attain this goal, the coordination of the activities in SC is mandatory. Vehicle routing problem (VRP) is one of the main combinatorial optimization problems in the SC. The VRP was introduced by Dantzig & Ramser (1959). It has several variants based on its characteristics, and capacitated VRP (CVRP) is one of them (see, e.g., Toth & Vigo, 2002)). The CVRP consists of determining vehicle routes through a set of geographically scattered customers, subject to the various constraints including the limitation of the capacity of the vehicles. The common objective of CVRP is to minimize the transportation cost in terms of travel distance or travel time.

Once the products are collected from the manufacturers or suppliers, the routed vehicles must return (may be at different times) to the collection centers which are generally called ‘depot’. Generally, these depots can have multiple doors to receive the collected products from the routed vehicles. Therefore, properly scheduling these vehicles to those doors at the depot is another key issue in the SC. The vehicle scheduling problem (VSP) leads to assigning the vehicles to doors at the depot and sequencing them to each and every door, in order to minimize the waiting time which causes the additional cost and increases the total cost. Moreover, the literature regarding the SC revealed that most of the studies focused on one problem, but it is recommended to deal with several problems together (Van Belle et al., 2012) and to include some of the operations at the depot with VRP (Buakum & Wisittipanich, 2019). Therefore, in this study, routing the vehicles to collect the products from suppliers and scheduling the vehicles to doors at the depot are simultaneously solved. Hence, the objective of this integrated VRP and VSP (refer in this study as **VR&SP**) is to minimize the total cost which contains the following components: *vehicle travelling cost* between suppliers and depot, *loading cost* at the suppliers, *vehicle waiting cost*, *unloading cost* at the depot and *vehicle operation cost*.

2. METHODOLOGY

2.1 Problem description

At the first phase of the integrated VR&SP, only the VRP is considered. Accordingly, the homogeneous vehicles (equal vehicle capacities) initiate the routes from the depot at time zero and visit all assigned suppliers. After loading all the products at the suppliers in the allotted routes, all the vehicles return to the depot and wait (at the parking area) until it is their turn to come to unload the products through the doors at the depot. It should be emphasized that, not only the cost components relevant to VRP, but also the time components such as *travelling time* and *loading time* are also calculated in each and every route. Fig. 1 depicts the process of CVRP.

At the second phase, only the VSP is taken into account. Based on the arrival time, the vehicles which arrive to the depot are assigned to the doors under the first come first serve (FCFS) basis. At the same time, the sequencing vehicles to each door at the depot are taken place.

In this phase, in addition to the cost components relevant to VSP, *waiting time*, *vehicle changeover time* and *unloading time* at the doors of the depot are also determined. Then the waiting time of the vehicles at the parking area near the depot is converted into cost such that a time unit is proportionally equal to 5 cost units. Fig. 2 portrays the process of VSP. Eventually, the sum of all the components of the cost is considered as the solution to the integrated VR&SP.

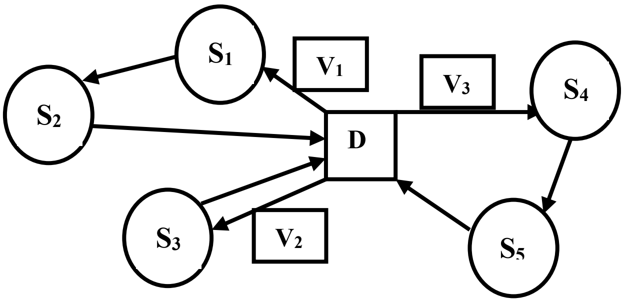


Fig. 1: Process of CVRP

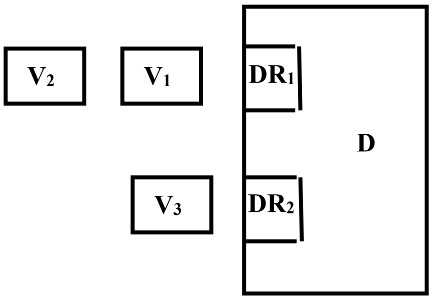
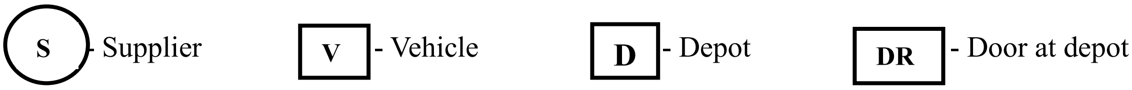


Fig. 2: Process of VSP



The assumptions in this study are as follows:

- Closed VRP and, vehicles are available at time zero
- Capacitated VRP with homogeneous fleets of vehicles
- Split supply is not allowed and each supplier must be served by exactly one vehicle
- Single depot with multi doors
- Changeover time at the doors of the depot is fixed for all the vehicles

2.2 Solution Method

A mixed integer quadratic programming (MIQP) model is developed to solve the integrated VR&SP. The Branch and Bound (B&B) algorithm is employed to obtain the exact solution to this MIQP using LINGO (version 18) optimization software. The programs are run on Intel Core i5 with 2.30 GHz CPU and 4 GB RAM. The input data for the small-scale instances of VR&SP are generated randomly based on the following parameters reported in Table 1 given below:

Table 1: Parameters of MIQP model of VR&SP

Parameters	**Distribution/ Value**	**Parameters**	**Distribution/ Value**
Travelling cost	Uniform (50, 200)	Travelling time	Uniform (20, 100)
Shipment	Uniform (20, 50)	Vehicle capacity	60 units
Unit loading cost	1 cost unit	Unit loading time	1 time unit
Unit unloading cost	1 cost unit	Unit unloading time	1 time unit
Preparation cost	10 cost units	Preparation time	10 time units
Vehicle operations cost	50	Vehicle changeover time	15 time units

3. RESULTS AND DISCUSSION

3.1 Small-scale instances of VR&SP

Since VRP is a NP-hard problem (Lenstra, J. K.; Rinnooy Kan, 1981), integrated VRP with VSP is also a NP- hard problem. Furthermore, LINGO is not capable to handle large-scale instances (as it always tries to obtain the exact optimum solution). Therefore, only the small-scale instances are taken into account to test the compatibility of the developed MIQP for VR&SP model. According to the parameters assigned in Table 1, fourteen small-scale test instances are generated, and the results of those instances are summarized in Table 2 below:

Table 2: Summary of small-scale instances of VR&SP

Instance No.	**No. of Suppliers**	**Total Shipments**	**Required No. of Vehicles**	**Allocated No. of Doors**	**Total Waiting Time (m)**	**Solution to VR&SP**
1	7	200	4	2	11	1347
2	8	220	4	2	50	1627
3	9	240	5	2	88	1717
4	10	260	5	2	183	2215
5	11	280	6	3	13	1952
6	12	300	6	3	38	2095
7	13	320	7	3	95	2456
8	14	340	7	4	171	2795
9	15	360	7	4	33	2572
10	16	380	8	4	26	2726
11	17	400	8	5	0	2703
12	18	420	8	5	21	2870
13	19	440	9	5	41	3134
14	20	460	9	5	73	3306

It should be noted that the size of the instances, in terms of the number of suppliers and number of doors at the depot, gradually increase as exhibited in Table 2. The total quantities that have to be collected from the number of suppliers with the required number of vehicles to the VRP are reported in Table 2 given above. In addition, in Table 2, the allocated number of doors at the depot of VSP is included. Also, Table 2 presents the total waiting time of the vehicles before unloading its shipments and the solutions to the VR&SP. As seen from Table 2, the feasibility of the developed MIQP model for the VR&SP is verified. The details of the instance of VRP with 7-suppliers and VSP with 2-doors at the depot (the first instance in Table 2) are described in the following subsection 3.2.

3.2 Route-wise results of the instance of VRP with 7-suppliers and VSP with 2-doors

This specific instance has 7-suppliers (S_1 to S_7) and 2- doors at the depot (DR_1 and DR_2). The route-wise details are illustrated in the following Table 3:

Table 3: Route-wise results of the instance of VRP with 7- suppliers and VSP with 2-doors

Vehicle V_1	**From**	**To**	**Arrival Time (Collected Products)**	**Vehicle V_2**	**From**	**To**	**Arrival Time (Collected Products)**
	D	S_6	64 (32)		D	S_3	52 (30)
	S_6	S_2	123 (54)		S_3	S_4	136 (56)
	S_2	D	**248** (54)		S_4	D	**267** (56)
Vehicle V_3	**From**	**To**	**Arrival Time (Collected Products)**	**Vehicle V_4**	**From**	**To**	**Arrival Time (Collected Products)**
	D	S_1	27 (29)		D	S_7	55 (33)
	S_1	S_5	97 (57)		S_7	D	**153** (33)
	S_5	D	**206** (57)				

It can be interpreted from Table 3 given above that the first route by the vehicle V_1 first visits the supplier S_6 from the depot D in 64 time units and collects 32 units of products. Next V_1 visits the supplier S_2 from S_6 in 123 time units (in total time from D) and added to 54 units of products (it contains the total products from both S_6 and S_2). Then V_1 returns to depot D from S_2 in 248 total time units with accumulated 54 units of shipments. Similarly, the details of the other routes by the vehicles V_2, V_3 and V_4 also can be interpreted from Table 3.

Table 4: Results of scheduling routed vehicles to doors at the depot

Door	Vehicle	Accumulated Shipments	Arrival Time (To depot)	Start Time (To unload)	Processing Time	Finish Time (To unload)
DR_1	V_4	33	153	153	48	201
DR_1	V_1	54	248	248	69	317
DR_2	V_3	57	206	206	72	278
DR_2	V_2	56	267	278	71	349

According to FCFS policy and as per the arrival time reported in Table 4, vehicle V_4 (the first vehicle to arrive at depot D in 153 time units with 33 units of products) is assigned to the door DR_1. With the changeover time of 15 time units, 48 (since *processing time = changeover time + accumulated shipments*) time units are necessary to completely unload the shipments and the job can be finished in 201 time units. Next, the second vehicle V_3 arrives in 206 time units and it is assigned to the door DR_2. Then, the third vehicle V_1 arrives in 248 time units which is assigned to the door DR_1 as it is the only free door at that time. Finally, the fourth vehicle V_2 arrives in 267 time units, but both doors are occupied by both vehicles V_3 and V_1 and therefore, it has to wait until the vehicle V_3 finishes unloading its shipments at 278 time units (it happens earlier than the finish time of V_1 which is 317 time units). Hence, the total waiting time by all 4 vehicles is 11 (=278-267) time units (in fact V_2 is the only vehicle which has to wait in this particular instance based on this assignment) and it is converted to 33 cost units (since it is assumed in this study that 1 unit of waiting time is equivalent to 3 cost units).

4. CONCLUSIONS/RECOMMENDATIONS

In this study, the VRP and VSP are solved simultaneously. In the integrated VR&SP model, the scheduling routed vehicles from the closed CVRP to the doors of the depot are taken place. A MIQP is developed to solve the VR&SP model using LINGO optimization software. The experimental results of randomly generated 14 small-scale test problems of VR&PS show the feasibility of the developed model. Therefore, it can be concluded that, this model is suitable for small-scale integrated VR&SP models. Since VR&SP is a NP-hard problem, heuristics or meta-heuristic methods are recommended to solve the large-scale instances of VR&SP model. Furthermore, this VR&SP model can be extended to VRP with cross-docking (CD) problems. Generally, CD centers have multi-doors in which some doors are assigned to receiving products from suppliers and some are assigned to shipping them to customers. Therefore, it is recommended that an integrated model combining VRP and VSP with CD can be attempted as a future study.

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