# A MATHEMATICAL MODEL FOR THE HARD TIME WINDOWS VEHICLE ROUTING PROBLEM WITH MOVING SHIPMENTS AT THE CROSS DOCK CENTER

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## INTRODUCTION

The main activities of warehousing in the traditional distribution centers of a supply chain (SC) are receiving, sorting, storing, order picking and shipping. Apte and Viswanathan (2000) stated that, these activities incur more than 30% of the total cost of the product. In the 1930s, to make SC very fast and productive, a new warehousing technique called "cross- docking (CD)" was introduced. Since the 1980s, after the successful application of this technique to Walmart food chain industry, it became popular among manufacturing and retailing companies. According to Vahdani and Zandieh (2010), up to 70% of the cost of warehousing can be reduced by implementing CD techniques in SC. Recently CD distribution techniques have attracted strong attention among researchers and practitioners. Vehicle routing plays a significant role to make SC more efficient. Since vehicle routing problem (VRP) plays an essential part in applying a cross-docking technique in the distribution centers, Lee, Jung, and Lee (2006) initiated the research on integrating vehicle routing problem with cross-docking (VRPCD). Thereafter, several researches have been conducted to solve VRPCD using different methods by incorporating different characteristics on VRP. However, the literature review of Dollaya and Warisa (2019) recommends that to focus on the activities inside at the cross-dock center (CDC) to make the VRPCD problem more realistic. Almost every related study in the literature does not consider the internal operations at CD.

Firstly in this study, *moving shipments* after unloading products from inbound vehicles at the receiving doors to shipping doors to uploading them to outbound vehicles is mainly taken into account. In addition to this, two more aspects are considered which are not being considered in the literature. Secondly, two different sets of vehicles with two different capacities are taken into account such that homogenous fleets of vehicles within pickup or delivery process and heterogeneous between pickup and delivery processes. Thirdly, loading and unloading shipments, in terms of both time and cost, at all the nodes including CDC is also taken into account. Moreover, the time windows (TW) characteristic are also added in the model for the problem of hard time windows vehicle routing problem with moving shipments at the cross-docking center (TW-VRPCD-MS). Therefore, the objective of this study is, by incorporating the above mentioned aspects, to minimize the total cost which includes transportation cost between nodes, cost of moving shipments inside CDC, vehicle operation cost and service cost at the nodes as well as at CDC while satisfying the time window of each node.

## METHODOLOGY

In this **TW-VRPCD-MS** problem, all the vehicles start their routes from CDC. In the ***pickup process***, from the suppliers using the fleet of homogeneous inbound vehicles, the orders of customers are collected and shifted to CDC to unload them at the indoors of CDC. Subsequently, those unloaded products at CDC are moved near the outdoors of CDC for ***consolidation process*** and subsequently they are loaded simultaneously to homogeneous, but with different capacity from inbound vehicles to outbound vehicles. In the ***delivery process***, the loaded products are delivered to relevant customers and all those outbound vehicles return to CDC at the end of their routes.

In both pickup and delivery processes, all the nodes (suppliers or customers) have their own ***time windows*** so that the vehicles must arrive at those nodes only in those particular time intervals. At the beginning, the randomly selected node (Among the suppliers in the pickup process or customers in the delivery process) is assigned to a vehicle which satisfies its time window. Now from the already selected node, if the vehicle does not exceed its maximum capacity, another randomly selected unvisited node is assigned to the same vehicle, with the satisfaction of the time window. If the vehicle exceeds its capacity, a new node has to be assigned to a new vehicle. In all three processes, once a node is reached, $A$ units of preparation time and $B$ units of time per pallet to load/ unload the products have to be considered in order to calculate the ***service time*** spent at each node. ***Arrival time*** of each vehicle from previous node to the new node is calculated when it reaches the new node after serving the previous node and travelling from previous node to the new node.

After reaching the receiving doors at CDC, the ***unloading time*** at receiving doors at CDC is calculated similar to service time described above. Unloaded products at CDC are moved from receiving doors to the shipping doors. Here, $B$ units of time per pallet is applicable to calculate the ***moving time*** from receiving doors to shipping doors and to calculate the ***ready time*** to start the loading at shipping doors at CDC. Demands of customers are consolidated according to their requests. Also in this case, the ***loading time*** at the shipping doors at CDC is applicable similar to the unloading time at the receiving door at CDC.

The factors considered for the total cost, which is to be minimized, are categorized as follows: ***Transportation cost*****:** Cost of travel in between nodes including CDC. ***Service cost at nodes*****:** The cost for loading or unloading products at the pickup or delivery nodes respectively. ***Service cost* at CDC:** The cost for unloading accumulated products by a vehicle (collected through a particular pickup route) at the receiving doors of CDC and cost for loading products by a vehicle (to be distributed in a particular delivery route) at the shipping doors of CDC. ***Moving Shipments cost*****:** The cost of moving the unloaded products from each inbound vehicle at the receiving doors of CDC to load them to the outbound vehicles at shipping doors of CDC. ***Vehicle operational cost*****:** The cost for maintaining/hiring the vehicles. It is noted that, service cost at CDC and cost of moving shipments inside CDC are considered as a cost due to the activities inside the CDC.

**Table 1: Input data for MINLP model of TW-VRPCD-MS**

| Input | Distribution/Value | Input | Distribution/Value |
|---|---|---|---|
| Travelling time | Uniform (20, 100) | Travelling cost | Uniform (50, 200) |
| Quantity | Uniform (20, 50) | Time horizon | 960 minutes |
| Preparation time | 10 units (minutes) | Unit shipping time | 1 unit (minute) |
| Preparation cost | 10 units (currencies) | Unit shipping cost | 1 unit (currency) |
| Inbound vehicle capacity | 80 | Outbound vehicle capacity | 50 |
| Operational cost of inbound vehicle | 150 | Operational cost of outbound vehicle | 100 |

A mixed integer nonlinear programming (MINLP) model is developed to solve the TW-VRPCD-MS problem. The problem is coded in LINGO (version 18) optimization software and solved using Branch and Bound (BB) algorithm. These programmes are run on Intel Core i5 with 2.30 GHz CPU and 4 GB RAM. The required input data of the randomly generated instances in small-scale of TW-VRPCD-MS are presented in Table 1:

## RESULTS AND DISCUSSION

### Computational Experiments of TW-VRPCD-MS

The optimal solution, number of nodes, required number of vehicles, total flow and the average computational time of 10 replicates of the same problem are presented in Table 2 given below as the results of the small size instances of TW-VRPCD-MS:

**Table 2: Results of small size instances of TW-VRPCD-MS**

| Problem No. | No. of Nodes | | Flow | No. of Vehicles Used | | Optimal Solution | Average Computational Time T (in s) |
|---|---|---|---|---|---|---|---|
| | Pickup | Delivery | | Inbound | Outbound | | |
| 01 | 02 | 03 | 100 | 02 | 03 | 2606 | 0.132 |
| 02 | 03 | 03 | 110 | 03 | 03 | 3186 | 0.164 |
| 03 | 03 | 04 | 120 | 03 | 04 | 3520 | 0.197 |
| 04 | 03 | 05 | 130 | 03 | 05 | 3806 | 0.240 |
| 05 | 04 | 05 | 140 | 02 | 05 | 3708 | 0.364 |
| 06 | 04 | 06 | 150 | 03 | 04 | 3887 | 0.750 |
| 07 | 04 | 07 | 160 | 03 | 04 | 4002 | 1.534 |
| 08 | 05 | 07 | 170 | 03 | 04 | 4120 | 3.123 |
| 09 | 06 | 07 | 180 | 03 | 06 | 4603 | 5.566 |
| 10 | 06 | 08 | 190 | 05 | 04 | 5063 | 11.591 |
| 11 | 07 | 08 | 200 | 03 | 05 | 4917 | 25.916 |
| 12 | 08 | 08 | 210 | 03 | 07 | 5733 | 66.355 |
| 13 | 08 | 09 | 220 | 03 | 05 | 4618 | 135.589 |
| 14 | 08 | 10 | 230 | 05 | 05 | 5589 | 316.818 |
| 15 | 09 | 10 | 240 | 03 | 05 | 5343 | 410.835 |
| 16 | 10 | 10 | 250 | 04 | 06 | 5683 | 661.932 |

The applicability of the proposed MINLP model can be observed from the results of the numerical experiments exhibited in Table 2. Since the average computational time is reasonably less for the above instances, this model can be used for last time planning for small scale problems with nodes up to 20.

**Convergence Analysis**

The plot of the average computational time T, against the total number ($x$) of the suppliers and customers as problem size is presented in Figure 1. It can be obtained from the fitted curve in Figure 1 that, the average computational time to obtain the optimal solution increases according to the exponential function, $T(x) = 0.002e^{0.63x}$ with the coefficient of determinant, $R$-squared value, as 98%. Hence, it can be stated that, the convergence rate of the problems considered in this study rises exponentially.

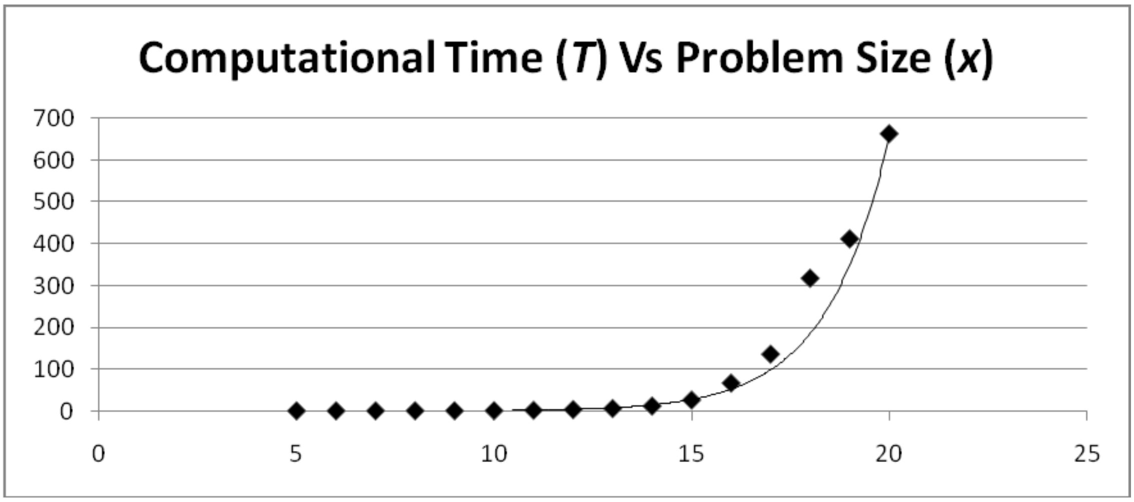


**Figure 1: Plot of Average Computational Time Vs Problem Size**

**CONCLUSIONS AND RECOMMENDATIONS**

Since the average computational time is reasonably less for the instances considered in this study, it can be concluded that this proposed model can be used for last time planning for similar size problems with nodes up to 20. Moreover, the convergence rate of the problems considered in this study is exponential. Thus, it can be concluded that when the number of nodes increases, consequently the computational time to reach the optimal solution increases exponentially. Therefore, this study recommends that, heuristic or metaheuristic methods are more appropriate to solve the medium scale problems with nodes between 20 and 50 and large scale problems with nodes more than 50 of TW-VRPCD-MS to obtain a near optimal solution in a moderately small computational time. Moreover, it is recommended for further studies to revise the proposed model according to the availability of vehicles for transportation, temporary storage capacity at CDC and budget allocated for transportation.

**REFERENCES**

Apte, U. M., Viswanathan, S., (2000). Effective cross docking for improving distribution efficiencies. *International Journal of Logistics*, 3 (3), 291–302. https://doi.org/10.1080/713682769

Dollaya, B., and Warisa, W., (2019). A literature review and further research direction in cross-docking. *Proceedings of the International Conference on Industrial Engineering and Operations Management 2019* (pp. 471-481). Bangkok , Thailand. http://www.ieomsociety.org/ieom2019/papers/121.pdf

Lee, Y. H., Jung, J. W., and Lee, K. M., (2006). Vehicle routing scheduling for cross-docking in the supply chain. *Computers & Industrial Engineering, 51(2)*, 247–256. https://doi.org/10.1016/j.cie.2006.02.006

Vahdani, B., and Zandieh, M., (2010). Scheduling trucks in cross-docking systems: Robust meta-heuristics. *Computers & Industrial Engineering,* 58(1), 12–24. https://doi.org/10.1016/j.cie.2009.06.006