I_P-SEPARATION AXIOMS IN IDEAL TOPOLOGICAL SPACES

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INTRODUCTION

Separation axioms play a major role in topological space analysis. The separation axioms deal with the use of topological concepts to distinguish among disjoint sets and distinct points. In topological spaces, there are many different forms of separation axioms. We primarily discuss T_0 space, T_1 space, and T_2space(Hausdorff space) spaces in topological spaces. Separation axioms between T_0 space, T_1 space, and T_2 space(Hausdorff space) are the focus of several topologists[1,2,3].
Several studies were conducted in the context of ideal topological spaces after the concept of ideals was introduced. In 1930, Kuratowski [4] and Vaidyanathasamy [5] were the first to bring up the concept of ideals in topological spaces. The separation axiom in ideal topological spaces was presented by Dontchev [2] in 1995, and it has since been improved by a number of scholars.
In this study, *Ip*-open sets are used to establish a new kind of separation axiom in ideal topological spaces, called *Ip*-separation axioms. In ideal topological space, this includes Ip-T_0 space, Ip-T_1 space, and Ip-T_2 space(Hausdorff space). The known implications of these axioms among themselves and with other axioms are examined, as well as some of their most important features.

METHODOLOGY

Definition 2.1 An ideal I on a topological space (X ,τ) is a nonempty collection of subsets of X which satisfies the following properties:
1) $A\in I$ and $B\subseteq A$ implies $B\in I$.
2) $A \in I$ and $B\in I$ implies $A\cup B\in I$.

If I is an ideal on X, then (X, τ, I) is called an ideal topological space.
Definition 2.2 A subset A of an ideal topological space (X, τ, I) is said to be *Ip*-closed set if $A^* \subseteq U$ whenever $A\subseteq U$ and U is preopen.
We define the complement of an *Ip*-closed set as being an *Ip*-open set.

Definition 2.3 Subspace ideal Topology
Let (X, τ, I) be an ideal topological space. If Y is a subset of X , then the collection $\tau_Y = \{I_0 \cup Y \mid I_0 \in I\}$ is an ideal on Y and by (Y,τ_Y , I_Y) is called the subspace ideal topology.
Definition 2.4 *Ip*-Irresolute Map
Let (X, τ, I) and (Y, σ, J) be two ideal topological spaces. A map $f : (X, \tau, I) \longrightarrow (Y, \sigma, J)$ is called *Ip*-irresolute if $f^{-1}(V)$ is a *Ip*-closed set in (X, τ, I) for every Ip-closed set V in (Y, σ, J).
Definition 2.5 An ideal topological space (X, τ, I) is said to be
1) Ig-T_0 space [7] if for each pair of distinct points x, y of X, there exists an *Ig*-open set containing one point but not the other .
2) Irg-T_0 space [8] if for each pair of distinct points x, y of X, there exists an *Irg*-open set containing one point but not the other .
3) αIg-T_0 space [9] if for each pair of distinct points x, y of X, there exists an αIg-open set containing one point but not the other .

Definition 2.6 An ideal topological space (X, τ, I) is said to be
1) Ig-T_1 space [7] if for each pair of distinct points x, y of X, there exists a pair of *Ig*-open sets, one containing x but not y and the other containing y but not x.
2) Irg-T_1 space [8] if for each pair of distinct points x, y of X, there exists a pair of *Irg*-open sets, one containing x but not y and the other containing y but not x.

3) αIg-T_1 space [9] if for each pair of distinct points *x*, *y* of *X*, there exists a pair of αIg--open sets, one containing *x* but not *y* and the other containing *y* but not *x*.

Definition 2.7 An ideal topological space (*X*, *τ*, *I*) is said to be

1) Ig-T_2 space [7] if for each pair of distinct points *x*, *y* of *X*, there exists an *Ig*-open sets *U* and *V* such that $x \in U$, $y \in V$ and $U \cap V = \Phi$.
2) Irg-T_2 space [8] if for each pair of distinct points *x*, *y* of *X*, there exists an *Irg*-open sets *U* and *V* such that $x \in U$, $y \in V$ and $U \cap V = \Phi$.
3) αIg-T_2 space [9] if for each pair of distinct points *x*, *y* of *X*, there exists an αIg-open sets *U* and *V* such that $x \in U$, $y \in V$ and $U \cap V = \Phi$.

RESULTS

3.1 I_P-T_0 Space

Definition 3.1.1 An ideal topological space (*X*, *τ*, *I*) is said to be Ip-T_0 space if for each pair of distinct
points *x*, *y* of *X*, there exists an *Ip*-open set containing one point but not the other.

Lemma 3.1.2 Every T_0 space in an ideal topological space is a Ip-T_0 space.

Proof. Let *x* and *y* be two distinct points in (*X*, *τ*, *I*) and *X* bea T_0 space. Then, there exists an open set *V* such that $x \in V$ and $y \notin V$. Since every open set is a *Ip*-openset, therefore *V* is an *Ip*-open set where $x \in V$ and $y \notin V$. This gives, (*X*, *τ*, *I*) is a Ip-T_0 space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.1.3 Consider the ideal topological space (*X, τ, I*) where X={*a*,*b*,*c*} with $\tau = \{X, \Phi, \{b,c\}\}$ and $I=\{\Phi,\{b\}\}$.Then, *X* is a Ip-T_0 space but not T_0 space. Since *b* and *c* are contained in all the open sets of *X*.

Lemma 3.1.4 Every Ip-T_0 space in an ideal topological space is a Ig-T_0 space.

Proof . Let *x* and *y* be two distinct points in (*X*, *τ*, *I*) and *X* be a Ip-T_0 space. Then, there exists an Ip-T_0 -open set *V* such that $x \in V$ and $y \notin V$. Since every *Ip*-open set is an *Ig*-open set, therefore *V* is an *Ig*-open set where $x \in V$ and $y \notin V$. This gives, (*X*, *τ*, *I*) is aIg-T_0 space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.1.5 Consider the ideal topological space (*X*, *τ*, *I*), where *X*={*a*,*b*,*c*} with $\tau=\{X,\Phi,\{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, *X* is aIg-T_0 space but not Ip-T_0 space. Since *b* and *c* are contained in all the *Ip*-open sets of *X*.

Lemma 3.1.6 Every Ip-T_0 space in an ideal topological space is a αIg- T_0 space.

Proof. Let x and y be two distinct points in (*X*, *τ*, *I*) and X be a Ip-T_0 space. Then, there exists an *Ip*-open set V such that $x \in V$ and $y \notin V$. Since every *Ip*-open set is an αIg-open set, therefore V is an αIg-open set where $x \in V$ and $y \in V$. This gives, (*X*, *τ*, *I*) is a αIg-T_0 space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.1.7 Consider the ideal topological space (*X*, *τ*, *I*), where *X*={*a*,*b*,*c*} with $\tau=\{X,\Phi,\{b\},\{a,c\}\}$ and $I=\{\Phi,\{b\}\}$. Then, *X* is a αIg-T_0 space but not Ip-T_0 space. Since *a* and *c* are contained in the *Ip*-open sets *X* and {*a*,*c*} of *X*.

Lemma 3.1.8 Every Ip-T_0 space in an ideal topological space is a Irg-T_0 space.

Proof. Let *x* and *y* be two distinct points in (*X*, *τ*, *I*) and *X* be a Ip-T_0 space. Then, there exists an *Ip*-open set *V* such that $x \in V$ and $y \notin V$. Since every *Ip*-open set is an *Irg*-open set, therefore *V* is an *Irg*-open set where $x \in V$ and $y \notin V$. This gives, (*X*, *τ*, *I*) is a Irg- T_0 space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.1.9 Consider the ideal topological space (*X*, *τ*, *I*) where *X*={*a*,*b*,*c*} with $\tau=\{X, \Phi, \{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, *X* is a Irg- T_0 space but not Ip-T_0 space. Since *b* and *c* are contained in all the *Ip*-open sets of *X*.

Theorem 3.1.10 An ideal topological space (X, τ, I) is an Ip-T_0 space if and only if Ip-closure of distinct points are distinct.
Proof. Let x and y be two distinct points in X and X be an Ip-T_0 space. Then, there exists an Ip-open set V such that $x \in V$ but $y \notin V$. Also $x \notin V^C$ and $y \notin V^C$ where V^C is a Ip-closed set in X. Since $cl_P(\{y\})$ is the intersection of all Ip-closed sets which contains y, $y \in cl_P(\{y\})$ but $x \notin cl_P(\{y\})$ as $x \notin V^C$. Thus, $cl_P(\{x\}) \neq cl_P(\{y\})$.
Conversely, suppose that for any pair of distinct points x and y in X, $cl_P(\{x\}) \neq cl_P(\{y\})$. Then, there exists at least one point $z \in X$ such that $z \in cl_P(\{x\})$ but $z \notin cl_P(\{y\})$. If $x \in cl_P(\{y\})$, $cl_P(\{x\}) \subset cl_P(\{y\})$, then $z \in cl_P(\{y\})$, which is a contradiction. Hence $x \notin cl_P(\{y\})$. Now, $x \notin cl_P(\{y\})$ implies $x \in (cl_P(\{y\}))^C$, which is an Ip-open set in X containing x but not y. Hence X is a Ip-T_0 space.
Theorem 3.1.11 Every subspace of a Ip-T_0 space is a Ip-T_0 space.
Proof. Let X be a aIp-T_0 space and Y be a subspace of X.Let x, y be two distinct points of Y. Since $Y \subseteq X$ and X is a Ip-T_0 space, there exists an Ip-open set V such that $x \in V$ but $y \notin V$. Then, there exists an Ip-open set $V \cap Y$ in Y which contains x but does not contain y. Hence Y is a Ip-T_0 space.
Theorem 3.1.12 Let $f : (X, \tau, I) \longrightarrow (Y, \sigma, J)$ be a Ip-irresolute bijective map. If Y is a Ip-T_0 space then X is Ip-T_0 space.
Proof. Assume that Y is a Ip-T_0 space. Let u, v be two distinct points of Y. Since f is a bijection, for every $x, y \in X$ such that $f^{-1}(u) = x$ and $f^{-1}(v) = y$. Since Y is a Ip-T_0 space, there exists an Ip-open set H in Y such that $u \in H$ but $v \notin H$. Since f is Ip-irresolute, $f^{-1}(H)$ is a Ip-open set in X containing $f(x) = u$ but not containing $f(y) = v$. Thus, there exists an Ip-open set $f^{-1}(H)$ in X such that $x \in f^{-1}(H)$ but $y \notin f^{-1}(H)$ and hence X is a Ip-T_0 space.

3.2 *Ip*-T_1 Space

Definition 3.2.1 An ideal topological space (X, τ, I) is said to be Ip-T_1 space if for each pair of distinct points x, y of X, there exists a pair of Ip-open sets, one containing x but not y and the other containing y but not x. That is, An ideal topological space X is a Ip-T_1 space if for any x, $y \in X$ with $x \neq y$, there exist an Ip-open sets G, H such that $x \in G$, $y \notin G$ and $y \in H$, $x \notin H$.
Lemma 3.2.2 EveryT_1 space in an ideal topological space is a Ip-T_1 space.
Proof. Let x and y be two distinct points in (X, τ, I) and X be a T_1space. Then, there exists a pair of open sets U, V in X such that $x \in U$ and $y \notin U$, $y \in V$ and $x \notin V$. Since every open set is an Ip-open set, therefore U and V are Ip-open sets where $x \in U$ and $y \notin U$, $y \in V$ and $x \notin V$. This gives, (X, τ, I) is a Ip-T_1 space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.2.3 Consider the ideal topological space (X, τ, I) where $X=\{a,b,c\}$ with $\tau=\{X,\Phi,\{b\},\{a,b\},\{b,c\}\}$ and $I=\{\Phi,\{b\}\}$. Then, X is a Ip-T_1space but not T_1 space. Since there is no open set containing b but not containing a.
Lemma 3.2.4 Every Ip-T_1space in an ideal topological space is a Ig-T_1 space.
Proof : Let x and y be two distinct points in (X, τ, I) and X bea Ip-T_1space. Then, there exists a pair of Ip-opensets U, V in X such that x ∈ U and y ∉ U, y ∈ Vand x ∉V. Since every Ip-open set is an Ig-openset, therefore U and V are Ig-open set where x ∈ U andy ∉ U, y ∈ V and x ∉V. This gives, (X, τ, I) is a Ig-T_1space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.2.5 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau =\{X, \Phi, \{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, X is aIg-T_1space but notIp-T_1space. Since there is no Ip-open set containing b but not containing c.
Lemma 3.2.6 Every Ip-T_1space in an ideal topological space is a αIg-T_1space.
Proof. Let x and y be two distinct points in (X, τ, I) and X be a Ip-T_1space. Then, there exists a pair of Ip-open sets U, V in X such that $x \in U$ and $y \notin U$, $y \in V$ and $x \notin V$. Since every Ip-open

set is an αIg-open set, therefore U and V are αIg -open sets where $x\in U$ and $y\notin U$, $y\in V$ and $x\notin V$. This gives (X, τ, I) is a αIg-T_1space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.2.7 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{X, \Phi,\{a,c\}\}$ and $I=\{\Phi,\{b\}\}$. Then, X is a αIg-T_1spacebut not Ip-T_1space. Since there is no Ip-open set containing a but not containing c.

Lemma 3.2.8 Every Ip-T_1space in an ideal topological space is a Irg-T_1space.

Proof. Let x and y be two distinct points in (X, τ, I), and X bea Ip-T_1space. Then, there exists a pair of Ip-opensets U,V in X such that x ∈ U and y ∉ U, y ∈ Vand x ∉ V. Since every Ip-open set is an Irg-open set, therefore U and V are Irg-open set where x ∈ Uand y ∉ U, y ∈ V and x ∉V. This gives, (X, τ, I), is a Irg-T_1space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.2.9 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{X, \Phi, \{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, X is a Irg-T_1 space but not Ip-T_1space. Since there is no Ip-open set containing b but not containing c.

Lemma 3.2.10 Every Ip-T_1space in an ideal topological space is a Ip-T_0 space.

Proof. Suppose X is a Ip-T_1space, then for distinct points$x$ and y in X, there exists an Ip-open sets G and H such that $x\in G$, $y\notin G$ and $y\in H$, $x\notin H$. Since $G\cap H = \Phi$, $x\in G$ and $y \in H$. Then,either $x\in G$, $y\notin G$ or $y\in H$, $x\notin H$. Thus, X is a Ip-T_0 space.

The converse of the above lemma need not be true in general. This can be seen in the following example:

Example 3.2.11 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{ X, \Phi,\{b,c\}\}$ and $I=\{\Phi,\{b\}\}$.Then, X is a Ip-T_0 space but not Ip-T_1space. Since for the distinct points b and c there exists a pair of Ip-open sets $\{c\}$ and $\{b,c\}$ one contain b but not c and the other containing both b and c.

Theorem 3.2.12 Every subspace of a Ip-T_1space is also Ip-T_1space.

Proof. Let X be a Ip-T_1space and let Y be a subspace of$X$. Let $x, y\in Y\subseteq X$ such that $x\neq y$. By hypothesis X is Ip-T_1space, hence there exists an Ip-open sets U, V in X such that $x\in U$ and $y\in V$, $x\notin V$ and $y\notin U$. By definition of subspace, $U\cap Y$ and $V\cap Y$ are Ip-open sets in Y. Further, $x\in U$, $x\in Y$ implies $x\in U\cap Y$ also $y\in V$, $y\in Y$ implies $y\in V\cap Y$. Thus, there exist an Ip-open sets $U\cap Y$ and $V\cap Y$ in Y such that $x\in U\cap Y$, $y\in V \cap Y$ and $x\in V\cap Y$, $y\notin U\cap Y$. Therefore, Y is a Ip-T_1space.

Theorem 3.2.13 Let $f : (X, \tau, I) \longrightarrow (Y, \sigma, J)$ be an injectiveand Y be a Ip-T_1space. If f is Ip-irresolute,then X is a Ip-T_1space.

Proof. Assume that Y is a Ip-T_1space. Let $x, y\in Y$ suchthat $x\neq y$. Then, there exists a pair of Ip-open sets$U$, $V\in Y$ such that $f(x) \in U$ and $f(y) \in V$, $f(x) \notin V$ and $f(y) \notin U$ which implies $x\in f^{-1}(U)$, $y\in f^{-1}(V)$ and $x\notin f^{-1}(V)$, $y\notin f^{-1}(U)$ Since$f$ is Ip-irresolute. Therefore, $f^{-1}(U)$ and $f^{-1}(V)$ are Ip-open sets in X. Thus X is a Ip-T_1space.

Theorem 3.2.14 An ideal topological space X is said to be Ip-T_1space if and only if every singleton subset of X is Ip-closed set.

Proof. Suppose that X is a Ip-T_1space and $a\in$ X. We shallprove that $\{a\}^C$ is an Ip-open set. Let $x\in\{a\}^C$. Then, $x\neq a$, so there exists an Ip-open setU_X such that $x\in U_X$ but $a\notin U_X$. That is, $x\in U_X\subseteq \{a\}^C$. Thus, $\bigcup \{x\} \subseteq\bigcup\{ G_X : x\in \{a\}^C\} \subseteq \{a\}^C$. That is, $\{a\}^C = \bigcup\{ G_X : x\in \{a\}^C\}$. Since $\bigcup\{G_X : x\in \{a\}^C\}$ is Ip-open, $\{a\}^C$ is Ip-open.

Conversely suppose that $\{a\}$ is a Ip-closed set for any $a \in X$. Let $x, y\in X$ with $x\neq y$. Then, $y\in \{x\}^C$ where $\{x\}^C$ is Ip-open and x ∈ $\{x\}^C$. Therefore, $x \in \{y\}^C$ where $\{y\}^C$ is Ip-open and $y\notin \{y\}^C$. Thus, X is a Ip-T_1 space.

3.3 *Ip*-T_2Space(*Ip*-Hausdorff)

Definition 3.3.1 An ideal topological space (X, τ, I), is said to be Ip-T_2 space(Ip-Hausdorff) if for each pair of distinct points x, y of X, there exists an Ip-open sets U and$V$ such that $x \in U$, $y \in V$ and $U \cap V = \Phi$.
Lemma 3.3.2 Every T_2 space in an ideal topological space is a Ip-T_2 space.
Proof. Let x and y be two distinct points in (X, τ, I), and X bea T_2 space. Then, there exists a pair of open sets U, V in X such that $x \in U$ and $y \in V$ and $U \cap V = \Phi$. Since every open set is an Ip-open set, therefore$U$ and V are Ip-open sets where $x \in U$ and $y \in V$ and $U \cap V = \Phi$. This implies (X, τ, I), is a Ip-T_2 space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.3.3 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{X, \Phi,\{b\},\{a,b\},\{b,c\}\}$ and $I= \{\Phi,\{b\}\}$. Then, X is a Ip-T_2space but not T_2 space. Since the intersection of open sets $\{b\}$ and $\{b,c\}$ is nonempty.
Lemma 3.3.4 Every Ip-T_2 space in an ideal topological space is Ig-T_2 space.
Proof. Let x and y be two distinct points in (X, τ, I), and X bea Ip-T_2 space. Then, there exists a pair of Ip-opensets U, V in X such that $x \in U$ and $y \in V$ and $U \cap V = \Phi$. Since every Ip-open set is an Ig-open set,therefore U and V are Ig-open sets where $x \in U$ and $y \in V$ and $U \cap V = \Phi$. This implies (X, τ, I), is a Ig-T_2 space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.3.5 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau =\{X, \Phi,\{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, X is a Ig-T_2 space but not Ip-T_2 space. Since the intersection of an Ip-open sets $\{b,c\}$ and X is nonempty.
Lemma 3.3.6 Every Ip-T_2 space in an ideal topological space is a αIg-T_2 space.
Proof. Let x and y be two distinct points in (X, τ, I), and X bea Ip-T_2 space. Then, there exists a pair of an Ip-open sets U, V in X such that $x \in U$ and $y \in V$ and$U \cap V = \Phi$. Since every Ip-open set is an αIg-open set, therefore U and V are αIg-open setswhere $x \in U$ and $y \in V$ and $U \cap V = \Phi$. Thisimplies (X, τ, I), is a αIg-T_2 space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.3.7 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau =\{X,\Phi,\{a,c\}\}$ and $I=\{\Phi,\{b\}\}$. Then, X is a αIg-T_2 space but not Ip-T_2 space. Since the intersection of Ip-open sets $\{a,c\}$ and X is nonempty.
Lemma 3.3.8 Every Ip-T_2 space in an ideal topological space is a Irg-T_2 space.
Proof. Let x and y be two distinct points in (X, τ, I), and X bea Ip-T_2 space. Then, there exists a pair of an Ip-open sets U, V in X such that $x \in U$ and $y \in V$ and $U \cap V = \Phi$. Since every Ip-open set is an Irg-open set, therefore U and V are Irg-open sets where $x \in U$ and $y \in V$ and $U \cap V = \Phi$. This implies (X, τ, I), is a Irg-T_2 space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.3.9 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{X,\Phi,\{b,c\}\}$ and $I=\{\Phi,\{a\}\}$. Then, X is a Irg-T_2 space but not Ip-T_2 space. Since the intersection of an Ip-open sets $\{b,c\}$ and X is nonempty.
Lemma 3.3.10 Every Ip-T_2 space in an ideal topological space is a Ip-T_1 space.
Proof . Suppose X is a Ip-T_2 space, then for distinct points x and y in X there exist an Ip-open sets G and H such that $G \cap H = \Phi$. Therefore, $x \in G$, $y \notin G$ and $y \in H$, $x \notin H$. Thus, X is a Ip-T_1space.
The converse of the above lemma need not be true in general. This can be seen in the following example:
Example 3.3.11 Consider the ideal topological space (X, τ, I), where $X=\{a,b,c\}$ with $\tau=\{X, \Phi,\{b\},\{a,b\},\{b,c\}\}$ and $I= \{\Phi,\{b\}\}$. Then, X is a Ip-T_1 space but not T_2 space. Since the intersection of an Ip-open sets $\{a,b\}$ and $\{a,c\}$ is nonempty.
Theorem 3.3.12 Every subspace of a Ip-T_2 space is also Ip-T_2 space.

Proof. Let X be Ip-T_2 space and let Y be a subspace of X.Let a, $b \in Y \subseteq X$ with $a \neq b$. By hypothesis,there exist an Ip-open sets G, H in X such that $a \in G$and $b \in H$, $G \cap H = \Phi$. By definition of subspace,$G \cap Y$ and $H \cap Y$ are Ip-open sets in Y. Further $a \in G$, $a \in Y$ implies $a \in G \cap Y$ and $b \in H$, $b \in Y$ implies $b \in H \cap Y$. Therefore, $G \cap Y$ and $H \cap Y$are disjoint Ip-open sets in Y such that $a \in G \cap Y$ and $b \in H \cap Y$. Thus, Y is a Ip-T_2 space.

Theorem 3.3.13 If $\{x\}$ is Ip-closed in X, for every $x \in X$, then X is Ip-T_2 space.

Proof. Let x, y be two distinct points of X such that $\{x\}$ and$\{y\}$ are Ip-closed. Then, $\{x\}^C$ and $\{y\}^C$ areIp-open in X such that $y \in \{x\}^C$ but $x \notin \{x\}^C$ and $x \in \{y\}^C$ but $y \notin \{y\}^C$. Thisimplies, $\{x\}^C \cap \{y\}^C = \Phi$. Hence X is Ip-T_2 space.

Theorem 3.3.14 If X is Ip-T_2 space, then for $y \neq x \in X$, thereexist an Ip-open set G such that $x \in G$ and $y \notin cl_P(G)$.

Proof. Let x, $y \in X$ such that $y \notin x$. Since X is Ip-T_2 space, there exist disjoint Ip-open sets G and H in X such that $x \in G$ and $y \in H$. Therefore H^C is Ip-closed set such that $cl_P(G) \subseteq H^C$. Since $y \in H$, we have $y \notin H^C$. Hence $y \notin cl_P(G)$.

Proposition 3.3.15 Let f, g: $(X, \tau, I) \longrightarrow (Y, \sigma, J)$ be Ip-continuous maps and Y be a Ip-T_2 space. Then, $\{x \in X \mid f(x) = g(x)\}$ is a Ip-closed set.

Proof. Let $A = \{x \in X \mid f(x) \neq g(x)\}$ and suppose $x \in A$.Since $f(x) \neq g(x)$, there are Ip-open sets U, V in Y such that $f(x) \in U$, $g(x) \in V$ and $U \cap V = \Phi$.Let $W = f^{-1}(U) \cap g^{-1}(V)$. Then, W is an Ip-openand $x \in W$. Moreover, $W \subseteq A$. Thus$A$ is an Ip-open, so $\{x \in X \mid f(x) = g(x)\}$ is a Ip-closed set.

Theorem 3.3.16 Let $f : (X, \tau, I) \longrightarrow (Y, \sigma, J)$ be a bijective,Ip-open map between the ideal topological spaces (X, τ, I) and (Y, σ, J) . If X is a T_2 space, then Y is a Ip-T_2 space.

Proof. Suppose that X is a T_2 space and let x, $y \in X$ be distinct points. Then, there exist disjoint open sets G, H in X, such that $x \in G$, $y \in H$. Since f is an Ip-open map, $f(x) \in f(G)$ and $f(y) \in f(H)$, where $f(G)$, $f(H)$ are Ip-open sets in (Y, σ, J). Now, $f(G) \cap f(H) = f(G \cap H) = f(\Phi) = \Phi$. Therefore Y is a Ip-T_2 space.

Theorem 3.3.17 Let $f : (X, \tau, I) \longrightarrow (Y, \sigma, J)$ be a one to one,Ip-continuous map between ideal topological space (X, τ, I) and (Y, σ, J) and Ybeing a T_2 space. Then, X is a Ip-T_2 space.

Proof. Let x, $y \in X$ be distinct points. Then $f(x)$, $f(y) \in Y$ are distinct points. Since Y is a T_2 space, thereexist disjoint open sets U, $V \in Y$ such that $f(x) \in U$, $f(y) \in V$. Then, $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Since f is Ip-continuous map, $f^{-1}(U)$ and $f^{-1}(V)$ are Ip-open sets. Now, $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\Phi) = \Phi$. Therefore, X is a Ip-T_2space.

CONCLUSIONS

The Ip-separation axiom, which has Ip-T_0 space, Ip-T_1 space, and Ip-T_2 space (Ip-Hausdorff space) was developed to define and explore the separation axioms in ideal topological spaces in a new way.In addition, we looked at certain important themes and their interconnections with other existing separation axioms.

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