# UNCONSTRAINED GRADIENT BASED OPTIMIZATION OF QUADRATIC FORM EQUATIONS


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## INTRODUCTION

Optimization influences every field such as Management, Mathematics, Accounting, Engineering and etc. Research uses either gradient or non-gradient based optimization methods for solving nonlinear forms of equation. However, research outcomes show that the gradient based methods give more accurate results than the other one. There are several optimization methods for handling constraint and unconstraint problems. For the purpose, several tools with optimization capabilities are available, namely, MS *Excel solver, MATLAB,MathCAD, Mathamatica, Maple* and etc. Those tools have been developed for a wide range of requirements using a specific algorithm. Sometimes it may not be suitable to handle the problem at hand. In particularly, fminuc- matlab function is used to handle unconstrained minimization class problems but it has several drawbacks; it can be used to execute one method at a time, need to pass method names as a parameter and it is not supported for alpha-finding method, it is difficult for analyzing or comparing output and finally it is a single function with limited features and it is difficult to understand by users.

To overcome these issues, in this research study, an attempt is made to develop a user friendly *MATLAB GUI* tool to handle a specific problem of solving unconstrained minimization problem of quadratic form equation. This tool is featured with six unconstrained gradient methods. It has two parts one for finding the solution of the given problem and other one for analyzing the methods for the particular problem. This tool will be useful for both undergraduate students and researchers who are handling the specific problems.

In this study, the principal objective is that of finding variance $x$ for which a given function $f(x)$ is minimized. It is true that a practical design problem would be rarely be unconstrained still, a study of this class of problems is important for the following reasons:

- The constraints do not have significant influence in certain design problems.
- Some of the powerful and robust methods of solving constrained minimization problems require the use of unconstrained minimization techniques.
- The studies of unconstrained minimization techniques provide the basic understanding necessary for the study of constrained minimization methods.
- The unconstrained minimization methods can be used to solve certain complex engineering analysis problems. For example, the displacement response (linear or nonlinear) of any structure under any specified load condition can be found by minimizing its potential energy. Similarly, the eigenvalues and eigenvectors of any discrete system can be found by minimizing the Rayleigh quotient.

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## METHODOLOGY

### Classification of Unconstrained Minimization Methods

Several methods are available for solving an unconstrained minimization problem. These methods can be classified into two broad categories as direct search methods and descent methods. The direct search methods require only the objective function values but not the partial derivatives of the function in finding the minimum and hence are often called the non-gradient methods. The direct search methods are also known as *zeroth-order* methods since they use *zeroth-order* derivatives of the function. These methods are most suitable for simple problems involving a relatively small number of variables.
These methods are, in general, less efficient than the descent methods. The descent techniques require, in addition to the function values, the first and in some cases the second derivatives of the objective function. Since more information about the function being minimized is used (through the use of derivatives), descent methods are generally more efficient than direct search techniques. The descent methods are known as gradient methods. Among the gradient methods, those requiring only first derivatives of the function are called first-order methods; those requiring both first and second derivatives of the function are termed second-order methods.

### Unconstrained Minimization Methods

### Descent methods

Steepest descent (Cauchy) method, Fletcher–Reeves method, Newton's method, Marquardt method, Quasi-Newton methods, Davidon Fletcher Powell method, Broyden Fletcher Goldfarb Shanno method

### General Approach

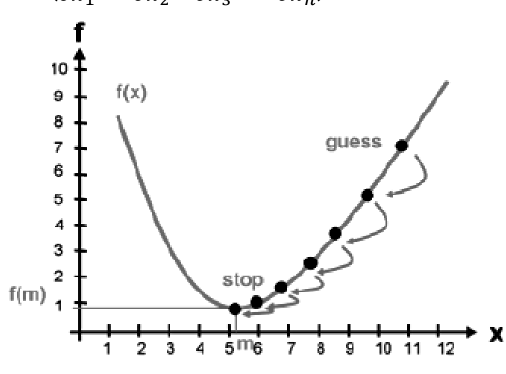
All the unconstrained minimization methods are iterative in nature and hence they start from an initial trial solution and proceed toward the minimum point in a sequential manner. The iterative process is given by $x_{i+1} = x_i + \lambda_i * s_i$, where $x_i$ is the starting point, $s_i$ is the search direction, $\lambda_i$ is the optimal step length and $x_{i+1}$ is the final point in iteration $i$. It is important to note that all the unconstrained minimization methods require an initial point $x_1$ to start the iterative procedure, and differ from one another only in the method of generating the new point $x_{i+1}$ (from $x_i$) and in testing the point $x_{i+1}$ for optimality.

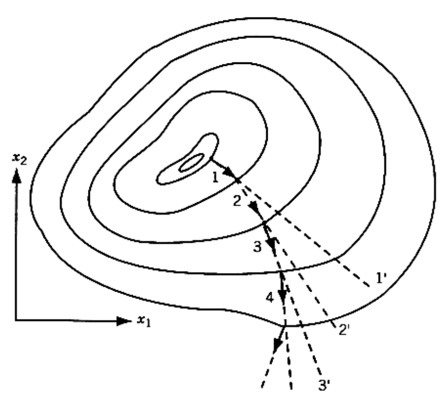
### Indirect Search (Descent) Methods

### GRADIENT OF A FUNCTION

The gradient of a function is an n-component vector given by

$$\Delta f_{n\times 1} = \left(\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \frac{\partial f}{\partial x_3} \cdots \frac{\partial f}{\partial x_n}\right)$$



The gradient has a very important property. If we move along the gradient direction from any point in n-dimensional space, the function value increases at the fastest rate. Hence the gradient direction is called the direction of steepest ascent. Unfortunately, the direction of steepest ascent is a local property and not a global one. This is illustrated in Figure below, where the gradient vectors $\nabla f$ evaluated at points 1, 2, 3, and 4 lie along the directions 11', 22', 33', and 44', respectively. Thus the function value increases at the fastest rate in the direction 11' at point 1, but not at point 2. Similarly, the function value increases at the fastest rate in direction 22' (33') at point 2 (3), but not at point 3(4). In other words, the direction of steepest ascent generally varies from point to point, and if we make infinitely small moves along the direction of steepest ascent, the path will be a curved line like the curve 1-2-3-4 in Figure given below

Since the gradient vector represents the direction of steepest ascent, the negative of the gradient vector denotes the direction of steepest descent. Thus any method that 3 makes use of the gradient vector can be expected to give the minimum point faster than one that does not make use of the gradient vector. All the descent methods make use of the gradient vector, either directly or indirectly, in finding the search directions. Before considering the descent methods of minimization, we prove that the gradient vector represents the direction of steepest ascent.

**Evaluation of the Gradient**

The evaluation of the gradient requires the computation of the partial derivatives $\frac{\partial f}{\partial x_i}, i = 1,2, \ldots n.$ There are three situations where the evaluation of the gradient poses certain problems (i) The function is differentiable at all the points, but the calculation of the components of the gradient, $\frac{\partial f}{\partial x_i}$, is either impractical or impossible (ii) The expressions for the partial derivatives $\frac{\partial f}{\partial x_i}$, can be derived, but they require large computational time for evaluation (iii) The gradient $\nabla f$ is not defined at all the points.

In the first case, the forward finite-difference formula,

$$\frac{\partial f}{\partial x_i}|x_m \simeq \frac{f(x_m + \nabla x_i u_i) - f(x_m)}{\nabla x_i}, i = 1, 2 \ldots, n$$

can be used to approximate the partial derivative $\frac{\partial f}{\partial x_i}$ at $x_m$. If the function value at the base point $x_m$ is known, this formula requires one additional function evaluation to find $\frac{\partial f}{\partial x_i}|x_m$. Thus it requires n additional function evaluations to evaluate the approximate gradient $\frac{\partial f}{\partial x_i}|x_m$. For better results we can use the central finite difference formula to find the approximate partial derivative $\frac{\partial f}{\partial x_i}|x_m$

$$\frac{\partial f}{\partial x_i}|x_m \simeq \frac{f(x_m + \nabla x_i u_i) - f(x_m - \nabla x_i u_i)}{2\nabla x_i}, i = 1, 2 \ldots, n.$$

**Newly developed Tool**

In the main window, if 'Unconstrained method' is selected, it will prompt another window where,

- User can insert a function in the given textbox labeled as '*function'*. (It'll be allowing only quadratic functions.)
- Then user can choose the *unconstrained methods*.
- User can give the initial values of the variables.
- User has to choose the step size and tolerance.
- Then user can get the output of whole selected methods with iterations results and its graphical representation.
- Last window will show all methods' output in the same table for comparison.

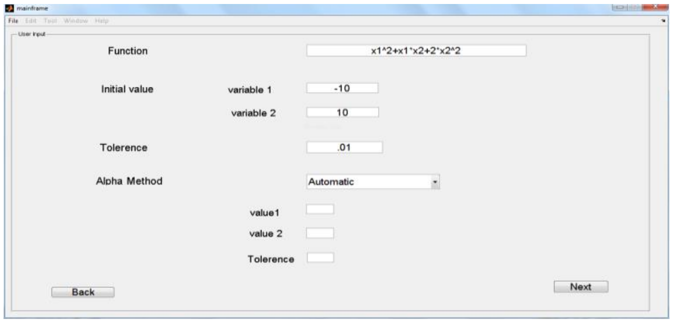



Figure 1 User Input

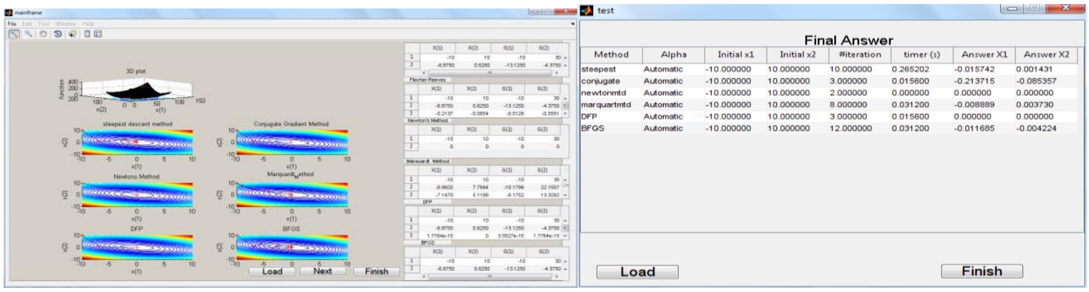



Figure 2 Comparison wind

The figure 1 shows input function ${x_1}^2 + x_1x_2 + {x_2}^2$ and tolerance 0.01 and initial values 10, -10 for $x_1, x_2$ respectively, with this selection of step size method as *Goldensection* and initial value for this step size method as 1 , -1 and tolerance as 0.01. In figure 3 shows the analysis part for whole six methods and its graphical outputs and analysis part for each method. This is easy to compare each method with figure 2 and suggest which method gives more accurate answer of the given problem.

## CONCLUSION

The newly developed MATLAB tool is successfully in solving the unconstrained optimization problem of functions of the quadratic form. The comparison of the methods has successfully been done by this tool. Any order of the function can be minimized and find the optimum using this tool because it converts the function to quadratic form and solves it based on gradient based optimization methods. In the comparison, user can analye and identify the suitable alpha finding method for a specific problem using the output table “Final Answer”. This tool is very useful for researchers to verify their results by comparing all the gradient based methods at a time and the undergraduates to understand and compare popular optimization methods.

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