# AN EMPIRICAL ANALYSIS OF SRI LANKAN EXCHANGE RATE CHANGES BY USING MARKOV CHAIN MODEL

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## INTRODUCTION

In the globalization era, the economic interdependence of the countries around the globe increases; it becomes necessary to understand the nature of exchange rate movements and its significant impact on countries performance with the rest of the world. Exchange rate plays a vital role in international trade and investment as they effect on prices of goods and services worldwide. An exchange rate is the price of one currency in terms of another currency. Therefore, movements in rate of exchange can have a significant impact on a particular country and its trading partners. Given the open, small and more import dependent nature of Sri Lankan economy will easily affected by exchange rate fluctuations. Further, forecasting the rate of exchange is very important to assess the benefits and risks attached to international as well as Sri Lankan business environment. However, the empirical studies on this topic have been increasingly evolved in recent literature. Few studies (Hooper and Marquez, 1995; Bernard and Jensen, 2004) provide evidences related to exchange rate movements and its effect on trade flows. Still predicting the exchange rate movement is an unresolved issue in finance literature. This study is tried to fill this gap by analyzing exchange rate movement of Sri Lankan rupee – with world leading currencies (US dollar, Euro, British pound) price changes by employing Markov chain model. Thus, the main purpose of this study is to determine the propensity of increasing and decreasing the exchange rate. Doubleday *et al.* (2011) have analyzed stock market price trends by determining probabilities of the market transitions between various states. Deju Zhang *et al.* (2009) have studied on forecasting the China's stock market trend based on Markov chain model approach. In Markov chain the outcome of an experiment depends only on the outcome of the previous experiment. Exchange rate also follows a random walk implies that the exchange rate changes are as independent of one another as the gain and losses. If we go through the application of Markov chain model, it will be useful to focus on understanding the usage of exchange rate. The exchange rate is used when simply converting one currency to another or for engaging in speculation or trading in the foreign exchange market. In this study the following objective was considered, to construct the two and four state Markov chain model for the movement of exchange rate data. Further, in order to estimate the transition probability matrix, the average transition periods and the prediction of the long run distribution for the exchange rate movement were used by the developed model.

## METHODOLOGY

Data for this research were retrieved from secondary source published by www.exchangerates.org.uk. Based on the availability and consistency of the data, daily exchange rate value of Sri Lankan rupees (LKR) versus world leading currencies: US dollar, Euro, British pound was collected. It covers the recent time period from 6th October 2009 to 24th November 2014 amounting to 1876 days. This study focuses on analysis of exchange rate using a discrete time stochastic model, namely a Markov chain. A stochastic process is a family of random variables $\{X(t), t \in T\}$, where the parameter "$t$" is running over a suitable index set $T$. The conditional distribution of $X(t_n)$ for given values of $X(t_1), X(t_2), \dots\dots, X(t_{n-1})$ dependence only on $X(t_{n-1})$ is called Markov dependents. A discrete or continuous parameter stochastic process display the property of Markov dependents is called Markov process. A special kind of Markov process is a Markov chain. The discrete state space Markov process is called Markov chain. The conditional probability $P(X_{n+1} = j | X_n = i) = p_{ij}$ is called the first step transition probability. For a

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finite state Markov chain $\{X_n\}$ the transition probability $p_{ij}$ can be represented by a matrix which is called transition probability matrix. With these transition probabilities, a $k \times k$ matrix, $P = (p_{ij})$, called the first step transition probability matrix of the Markov chain.

$$P = (p_{ij}) = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}$$

Each row of $P$ is the probability distribution relating to a transition from state $i$ to state $j$ and with the following properties: $0 \le p_{ij} \le 1$ for all, $i, j$ and $\sum_{j=1}^{k} p_{ij} = 1$, for all $i$.

A Markov chain is to have a steady state probability distribution if there exists a vector $\pi$ such that given a transition probability matrix $P$: $\pi = \pi P$.

If all states of a chain communicate and are not periodic, then the chain is said to be ergodic.

If a finite Markov chain is ergodic then

$$\lim_{n\to\infty} P^n = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_k \\ \pi_1 & \pi_2 & \cdots & \pi_k \\ \cdots & \cdots & \cdots & \cdots \\ \pi_1 & \pi_2 & \cdots & \pi_k \end{bmatrix}$$

where $\pi = (\pi_1, \pi_2, \ \ldots, \ \pi_k)$ with $0 < \pi_j < 1$ and $\sum_{j=1}^{k} \pi_j = 1$.

This steady state probability vector $\pi$ can be viewed as the unique distribution of a random variable in the long-run. Also mean recurrent times $m_{jj}$ are obtained by $m_{jj} = 1/\pi_j$, for every $j$.

## MODEL CONSTRUCTION

The data were arranged into two models of analysis such as two, four state Markov chain model and each study will be considered separately. Let $Y_n$ be the rupees of exchange rate on $n^{th}$ day. Then the random variable $D_n$ denoted by: $D_n = Y_n - Y_{n-1}$.

**Case-1:** Each day was classified as indicating exchange rate rupee higher or lower than the previous day for the financial market, considering the movement from a category of jump up or jump down on a day to a category of jump up or jump down the next, thus letting classification of two states, namely:

State 1 (jump up): Today's exchange rate rupee is greater than the exchange rate rupee of the previous day

State 2 (jump down): Today's exchange rate rupee is less than or equal to exchange rate rupee of the previous day

A sequence of daily changes on the state of the system may be able to form a binary random variable $X_n$ denoted by

$$X_n = \begin{cases} 1, & \text{if } D_n > 0, \\ 2, & \text{if } D_n \le 0. \end{cases}$$

Therefore above random variable $\{X_n\}$ is known as a Markov chain with state space $\{1, 2\}$.

**Case-2:** Based on the case-1 model, jump up and jump down were each partitioned into two subcategories each, namely, small and large. Transitions for this case possessed of moving from a category of jump up or jump down one day to a category of jump up or jump down the next, namely:

State 1: Large jump up (jump up greater than or equal to "$a$" rupees)

State 2: Small jump up (jump up between 0 and "$a$" rupees)

State 3: Small jump down (jump down less than or equal 0 and greater than "$-a$" rupees)

State 4: Large jump down (jump down less than or equal to "$-a$" rupees)

In this case state of the system denote the random variable $X_n$ as

$$X_n = \begin{cases} 1, & \text{if } \quad D_n \ge a, \\ 2, & \text{if } \quad a > D_n > 0, \\ 3, & \text{if } \quad 0 \ge D_n > -a, \\ 4, & \text{if } \quad D_n \le -a. \end{cases}$$

Here "$a$" is denoted as threshold value of absolute exchange rate changes and it was fixed by determining the absolute mean of the exchange rate daily changes of each currency separately.

## RESULTS AND DISCUSSION

The present study is based on time series data related to daily exchange rate. Hence, randomness of the data set is checked by using the following plots.

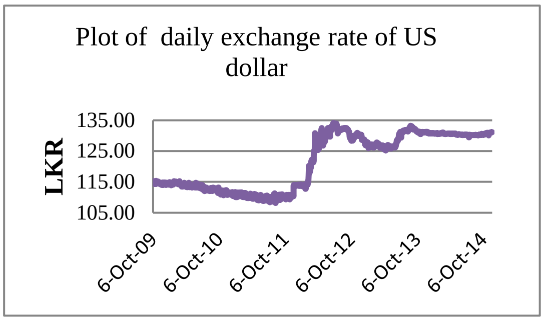


**Figure 1. Plot of daily exchange rate of US dollar**

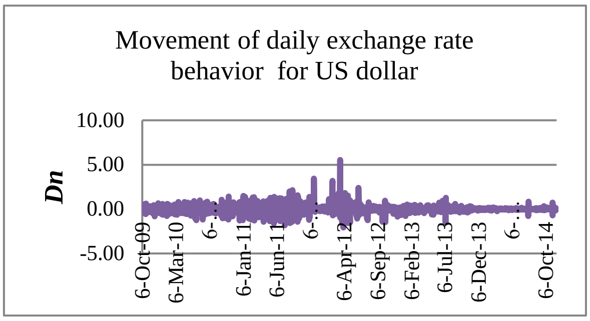


**Figure 2. Movement of daily exchange rate behavior for US dollar**

Figure 1 indicates that daily LKR exchange rate of US dollar has been changed over time. Figure 2 seem that there is no specific pattern in the movement of daily exchange rate and so that it can be applied to the random walk model for this study. The method of maximum likelihood has been used to estimate the transition probabilities under the certain assumptions. Thus the estimated transition probability $\hat{p}_{ij} = n_{ij}/n_i$, where $n_{ij}$ denotes the number of transitions from state $i$ to state $j$ and $n_i = \sum_j n_{ij}$. The transition probability matrix, steady state distribution and mean return times are estimated as follows, which takes into account the data for US dollar for two cases separately: The transition matrix for case-1was found to be:

$$P_1 = \begin{array}{c} \text{State} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.4259 & 0.5741 \\ 0.5513 & 0.4487 \end{bmatrix} \end{array} \quad \begin{array}{l} \pi = [0.4899, \quad 0.5101] \\ m = [2.0412, \quad 1.9604] \end{array}$$

Each row of this matrix $P_1$ is a probability vector and it is estimated probability for change in the behavior of the exchange rate movement for two consecutive days. In addition, matrix indicates that a given day irrespective of being in either state, there is a greater chance of transitioning to a reverse state. For example, estimate transition probability is interpreted as 57.41 % of the days, where exchange rate of US dollar that jump up will jump down. All states are communicated and aperiodic, then chain is an ergodic chain. Therefore, $\pi_1 = 0.4899, \pi_2 = 0.5101$, express that in a large number of days 48.99 % of the time the exchange rate change is predicted to tend to a jump up state and 51.01 % of the time price change is predicted to tend to a jump down state. Further, mean recurrence time vector specifies that the average return days of jump up state is ($m_{11} \cong 2$ days) approximately equal to jump down state ($m_{22} \cong 2$ days).

For the case-2, based on the data, compute the state threshold value "$a$" is 0.2317 rupees, then the transition matrix was found to be:

$$P_2 = \begin{array}{c} \text{State} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.1661 & 0.2102 & 0.3593 & 0.2644 \\ 0.1108 & 0.3387 & 0.4189 & 0.1316 \\ 0.1287 & 0.4342 & 0.3450 & 0.0921 \\ 0.3272 & 0.1949 & 0.2978 & 0.1801 \end{bmatrix} \end{array}$$

$\pi = [0.1574, \quad 0.3324, \quad 0.3650, \quad 0.1451]$ and $m = [6.3532, \quad 3.0084, \quad 2.7397, \quad 6.8918]$.

In matrix $P_2$ is a specific value of proportion for change in the behavior of the exchange rate movement in two successive days. First two row vectors observe that in a given day irrespective of being in either large jump up or small jump up state, there is a higher chance of transitioning to a state of small jump down than the other states. The third row vector indicates that there is a greater chance of transitioning to a state small jump up than other states. If a given day is in large jump down state, there is a better chance of transitioning to a state of large jump up than other states. Further, steady state probability $\pi$ is expressed as exchange rate movement among states is, after long days; consist of the following proportions in each category: 15.74 % in the state-1, 33.24 % in the state-2, 36.50 % in the state-3 and 14.51 % in the state-4. Finally, the average return period of the corresponding states 1, 2, 3, and 4 are $[6.3532, \quad 3.0084, \quad 2.7397, \quad 6.8918]$ days respectively.

**Table 1.** The matrix and vector estimators of the Euro and British pound currencies

| | | |
|---|---|---|
| Euro | $\begin{array}{c c} \text{State} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.4725 & 0.5275 \\ 0.4791 & 0.5209 \end{bmatrix} \end{array}$ <br> $\pi = [0.4760, \quad 0.5240]$ <br> $m = [2.1008, \quad 1.9084]$ <br> $a = 0.5546$ | $\begin{array}{c c} \text{State} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.2233 & 0.2642 & 0.2987 & 0.2138 \\ 0.1623 & 0.3019 & 0.3874 & 0.1483 \\ 0.1456 & 0.3498 & 0.3769 & 0.1276 \\ 0.1830 & 0.2618 & 0.3060 & 0.2492 \end{bmatrix} \end{array}$ <br> $\pi = [0.1703, \quad 0.3057, \quad 0.3548, \quad 0.1692]$ <br> $m = [5.8720, \quad 3.2712, \quad 2.8185, \quad 5.9102]$ |
| British pound | $\begin{array}{c c} \text{State} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.4725 & 0.5275 \\ 0.4417 & 0.5583 \end{bmatrix} \end{array}$ <br> $\pi = [0.4557, \quad 0.5443]$ <br> $m = [2.1944, \quad 1.8372]$ <br> $a = 0.6101$ | $\begin{array}{c c} \text{State} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.2290 & 0.2522 & 0.3333 & 0.1855 \\ 0.1654 & 0.3012 & 0.3720 & 0.1614 \\ 0.1599 & 0.2767 & 0.3977 & 0.1657 \\ 0.2202 & 0.2324 & 0.3456 & 0.2018 \end{bmatrix} \end{array}$ <br> $\pi = [0.1847, \quad 0.2711, \quad 0.3698, \quad 0.1745]$ <br> $m = [5.4142, \quad 3.6887, \quad 2.7042, \quad 5.7306]$ |

## CONCLUSIONS

In this study, daily exchange rate movement of Sri Lankan rupee with respect to US dollar, Pound sterling and Euro is empirically investigated by using a Markov chain model. The findings revealed that the daily exchange rate movement pattern will show a great propensity to have small jump up and small jump down for each currency in the four state models. Further, the result shows that the pattern of exchange rate movement is similar against both currencies such as Euro and British pound. The movement/fluctuation of exchange rate is generally subject to an exposure to various economic factors, for example, GDP of the country, interest rate, foreign capital flows, inflation and so on. However, still, there is no unique model can accurately predict all these changes and their impacts on daily exchange rate movement. Thus, Markov model also no exception. The model described here is used only to predict pattern of exchange rate movement. The same Markov chain frame-work can be further formulated to forecast amounts of exchange rate value too. Finally, the results of this study will definitely helpful for investors and policy makers and enable to design the exchange rate policy appropriately in the country.

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