A MATHEMATICAL MODEL TO CALCULATE THE SAFE DISTANCE BETWEEN PROGRESSIVE VEHICLES TO AVOID REAR END COLLISIONS

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INTRODUCTION

Road accidents which are caused by rear-end collisions are now increasing in numbers. Reason for most of such accidents is the driver's negligence that they do not keep a safe distance between the front vehicles. According to the census records of the Traffic Division of Sri Lanka Police, the 12.9 percent of the fatal accidents are caused by rear-end collisions. Out of these accidents 39.2 percent are injury free.

Therefore it is clear that a considerable amount of damages can be avoided by reducing rear end collisions of vehicles. To achieve that, it is required to maintain a safer distance between vehicles. Experienced drivers may have an idea about the gap between vehicles to avoid accidents, but it cannot be applied to all drivers and all types of vehicles. Therefore it is required to alert the driver from a system installed in the vehicle to avoid being too close to the front vehicle. This research has been conducted to develop a mathematical model to calculate the safe distance between vehicles by considering the relative motion, road condition, traffic on the road.

METHODOLOGY

Construction of mathematical model

When developing the mathematical model, three parameters have been identified as the key factors governing the alert time while driving. Those are,

1. Reaction time of the driver
2. Speed of the vehicle
3. Distance between the vehicles.

Derivation of safe distance between vehicles

To determine the safe distance we define two vehicles A and B running in the same direction one after the other at velocities of V and U respectively. Assume that A is leading. Since we are concerned of the rear vehicle the relative velocity with reference to the vehicle B can be given by equation (1).

$$V_{(A,B)} = V - U \qquad (1)$$

A collision is possible only if U is greater than V. This could be caused by either acceleration of vehicle B or deceleration of vehicle A. In case where vehicle A starts to decelerate at $t = t_1$ and stops at $t = t_4$. Assuming vehicle B detects the front vehicle deceleration and starts decelerating at $t = t_2$ and stops at $t = t_3$. The deceleration rates of vehicle A and B can be given by following equation (2) and (3) respectively. The deceleration rate is assumed to be constant over the time.

$$\frac{V - 0}{t_4 - t_1} = a \qquad (2)$$

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$$\frac{U-0}{t_3-t_2}=b \quad (3)$$

The safe distance (d) can be defined as,

(Travel distance of vehicle B from t = 0 to t = t_4) – (Travel distance of vehicle A from t = 0 to t = t_3)

$$d=\left[Vt_2+\frac{1}{2}\frac{U}{(t_3-t_2)}(t_3-t_2)^2\right]-\left[Vt_1+\frac{1}{2}\frac{V}{(t_4-t_1)}(t_4-t_1)^2\right]$$

$$d=\frac{(t_2+t_3)U}{2}-\frac{(t_1+t_4)V}{2} \quad (4)$$

Substituting (2) and (3) in (4) gives,

$$d=0.5\left(\frac{U^2}{b}-\frac{V^2}{a}\right)+Ut_2-Vt_1 \quad (5)$$

Since the vehicle B can only measure the relative velocity and its own velocity the equation (5) can be written in terms of U and $V_{(A,B)}$ .Using (1) and (5),

$$d=0.5\left(\frac{U^2}{b}-\frac{(U+V_{(A,B)})^2}{a}\right)+U(t_2-t_1)+V_{(A,B)}t_1 \quad (6)$$

Relative velocity calculation can be expressed in terms of time and distance measurements. The distance sensors of the rear vehicle measure and the distance to the forward vehicle at two consecutive instances. The relative velocity will be the ratio of distance travelled during the two time instances to the time elapsed. This process is continuously done in order to detect the relative velocity change which indicates a possibility of a collision.

Setting parameters of the safe distance formula

The time taken to apply brakes after hearing the alert is the reaction time of the driver and it is used as 1.5 s (National Highway Traffic Safety Administration (NHTSA), 1998). Most researches state that 90 percent of the drivers are decelerating the vehicles at a rate of 3.4 ms^{-2} (Officials, 2001) . Also it is acceptable that the rate of deceleration is comfortable for the drivers to stay within the driving lane and maintain the steering control on a wet surface also. For design purpose we assume the forward vehicle deceleration as 4.5 ms^{-2} which is higher than that of normal value. Therefore testing parameters for the model are listed as follows,

a = 3.4 ms^{-2}, b = 4.5 ms^{-2}, $(t_2 – t_1)$ = 1.5 s, t_1 = 1.5 s, initial distance travelled d_0 = 5 m

RESULTS AND DISCUSSION

The model itself gives the safe distance at any given speed of the vehicle. The parameters have been set to meet the critical condition by considering the extreme values. This model has been tested using a simulation tool by manually applying distance values and time values as given by the sensors in the real situation. Table 1 shows the data set used to test the model. The data set includes four consecutive distance measurements. The velocity of the vehicle is assumed to be at 18 ms^{-1} = 64.8 kmh^{-1}.

Table 1. Sample data set for testing the model

Test case \ Data	U (ms^{-1})	d1 (m)	d2 (m)	d3 (m)	d4 (m)
Vre1< Vre2	18	101	102	101	103
Current D > Safe D	18	102	103.6	103.8	104.8
Current D < Safe D	18	102	103.6	99.8	100
Current D > Critical D	18	29.4	32	33	34
Current D < Critical D	18	19	21	22	23

When the data is applied to the model the safe distance is calculated and it can be given to generate an alert by the system. Table 2 shows the calculated values for the given data.

Table 2. Test results

Test Case \ Result	Vre1	Vre2	d4 measurement	Warning Alarm Distance	Calculated critical distance	Critical Distance	Critical Alarm
Vre1< Vre2	5	10	0	0	7.536	0	No
Current D > Safe D	8	5	104.8	0	28.37	0	No
Current D < Safe D	8	1	100	100	41.04	41.04	No
Current D > Critical D	13	5	34	34	28.37	28.37	No
Current D < Critical D	10	5	23	23	28.37	28.37	Yes

It is clear that the warning alert will be activated when the distance is less than the safe distance and the critical alarm will be activated when the distance is less than the critical distance. According to the safe distance defined by NHTSA the test results gives a valid warning to prevent collisions. Table 3 gives the recommended safe distance data.

Table 3. Recommended safe distances by NHTSA (National Highway Traffic Safety Administration (NHTSA), 1998)

Speed (km/h)	Thinking Distance (m)	Braking Distance (m)	Safe distance (m)
40	27.8	18.4	46.2
50	34.8	28.7	63.5
60	41.7	41.3	83
70	48.7	56.2	104.9
80	55.6	73.4	129
90	62.6	92.9	155.5

CONCLUSIONS/RECOMMENDATIONS

The tested mathematical model gives the correct indication of alerts when the vehicles are running too close to each other. Therefore this model can be used to warn the drivers at correct times to prevent rear-end collisions. Important point of this model is it can update the safe distance by considering the current vehicle speed. Therefore it is suitable for any driving condition. As an example the safe distance would be very small during a town run and will be higher during a long distance journeys.

In order to test the model in real time, it is required to implement the model in an embedded system. Then it can be supplied with real sensor data and speed transducer data. After verifying the model in an embedded system it can be fitted to a real vehicle and test for accuracy. In the next phase of this research the model will be tested using real sensor data.

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